

6 Vertical Integration and Assurance of Markets

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1 INTRODUCTION

Four incentives for the vertical integration of firms have frequently been mentioned in the literature. Mergers may result from market power in either the primary-resource, intermediate-product or final-product markets.¹ Technological advantages accruing to combination can arise through increasing returns,² information advantages³ or decreased transactions costs, when firms place themselves in a co-operative rather than an adversarial relationship.⁴ Tax avoidance provides a third reason for integration.⁵ More generally, integration opens up a wider range of strategies in the face of regulation and more flexibility in implementing them. Finally, imperfections in the market for the intermediate product may lead firms to combine in order to bypass these problems by transferring goods internally.⁶ This chapter addresses the last of these issues. In particular, it studies the problem of price inflexibility in an intermediate-product market which is beset by stochastic demands, and the temporary shortages and gluts of this product that result. We hypothesise that firms choose to integrate if the expected profit from doing so exceeds that of the separate divisions acting independently. Both descriptive and normative conclusions regarding such an industry are drawn on the basis of the model presented.

The literature concerned with the behaviour of firms in industries with rigid prices has been growing rapidly.⁷ However, the effect of imperfect price-flexibility on the structure of markets has not been

addressed. General equilibrium models with rigid prices have been explored for the purpose of uniting the Keynesian and Walrasian models.⁸ Here also, the scope of activity for each firm has been taken as a datum rather than as a variable of the system.

The lack of treatment received by internal organisation in an explicit equilibrium framework is traceable to the fact that there is no incentive for either integration or divestiture in the static, perfectly competitive model.⁹ This allowed the specification of a fixed set of active, or potentially active, firms *ab initio*, describing each only by a set of technological possibilities. As interest developed in various generalisations, the resulting forces for realigning firms in ways other than that permitted by the initial listing were noticed. But a unified treatment that links the results of such actions to the underlying movement of the system towards equilibrium has yet to emerge. This chapter is an attempt to construct such a model, and it is hoped that the techniques and framework employed will find application beyond the narrow question addressed here.

The complexity and multiplicity of forces favouring or opposing alternative forms of internal organisation for the firm, make a general theory of vertical integration intractable. In concentrating on the effect of market imperfections on integration, we shall deliberately abstract from the first three of the incentives for mergers mentioned above. Firms in our model will behave competitively, will operate in unregulated, untaxed industries, and will produce according to a constant-returns-to-scale technology. Such firms, will derive an advantage from integration in the absence of some adverse technological consequences of this action. Since they always have the option of duplicating the behaviour they would have followed separately, integration can hardly be harmful to such firms. Therefore, a theory of integration in the absence of taxes on the organisational form of firms must at some stage rest on the existence of decreasing returns to an expansion of the scope of activities carried out by a single firm – or else integration will always be complete. In the relatively simple model presented below, the market advantages accruing to integration face countervailing forces in the shape of the decreased technological efficiency of the larger enterprise.¹⁰

The issue of vertical integration, at least in the USA, has arisen largely as a policy question. Should a vertical merger be permitted between two firms who actually desire to take such an action? This rather obvious fact has implications for public policy in the analysis of such cases. Although previous market-oriented studies of vertical

integration trace out its effects on prices, quantities, inventory levels and productive efficiency, they hypothesise the merger of two firms without relating that hypothesis to the value which such a move has to the firms in question.¹¹ Neither do they explore whether the factors causing the initial integration persist and will cause other similar firms to take the same step. In this chapter, we shall treat the integration-divestiture process as the result of an explicit equilibrating mechanism. We therefore focus on the issue of permitting or disallowing those mergers which are actively being pursued by firms in the industry, rather than on whether merger *per se* is socially valuable under arbitrary circumstances.

In Section II the basic model is presented. Section III is concerned with characterising the equilibria that can arise in this industry. Welfare and policy implications are developed in Section IV. Section V discusses some generalisations of the model, and a brief conclusion follows.

II THE MODEL

(1) STRUCTURE OF THE INDUSTRY

We construct a model in which there are three types of firm, called upstream, downstream and integrated. Upstream firms transform labour into an intermediate product using capital services in the process, according to the fixed-coefficient technology

$$x = \min \left(\frac{L}{a_L}, \frac{K}{a_K} \right). \quad (1)$$

Here, x is the output of intermediate product and L and K are the amounts of labour and capital used, respectively. Downstream firms use the intermediate product to produce a final output according to the production function

$$v = \min \left(\frac{y}{b_y}, \frac{K}{b_K} \right). \quad (2)$$

Here, y is the quantity of intermediate product used in this process, K is the quantity of capital used, and v is the quantity of final product produced.

Integrated firms use labour and capital in a two-stage production process. This yields final product as an output, but may involve the intermediate product as either a net input or a net output. The production function is

$$v = \min \left(\frac{z + \min \left(\frac{L}{a'_L}, \frac{K^1}{a'_K} \right)}{b'_z}, \frac{K^2}{b'_K} \right). \quad (3)$$

Here, z is the input of intermediate product in excess of the quantity the firm produces for its own use, which is given by the second term in the sum, and K^1 and K^2 are the quantities of capital allocated to the two stages of the production. We assume that the integrated firms have a technology inferior to their non-integrated counterparts, that is

$$a_L \leq a'_L; \quad a_K \leq a'_K; \quad b_y \leq b'_y; \quad b_K \leq b'_K.$$

Without a condition ensuring decreasing returns to integration, there is no reason for any producer to remain non-integrated. An analysis of equilibria in the form of organisation of firms in this industry can therefore be non-trivial only under such circumstances.

Since our concern is vertical integration of firms, attention will focus primarily on the market for the intermediate product. We suppose that prices are fixed. The price of labour will be taken to be unity; final product has a price p ; and the price of the intermediate product is q . The intermediate product market is subject to stochastic fluctuations in exogenous excess demand, and these lead to rationing of one side of the market or the other.

The industry we study is perhaps not the only user of the intermediate product. The random factors impinging upon this market result in a stochastic net demand (Z) for the intermediate product. Let

$$Z_+ = \max(0, Z) \text{ and} \\ Z_- = \min(0, Z)$$

and the distribution of Z be $f(\cdot)$.

The total demand for the intermediate product is given by the sum of the demand by downstream firms, by integrated firms (if they are net demanders), and the demanders outside this industry. These outside demands are comprised of non-stochastic components (D)

and (S) , and the stochastic net demands Z_+ . Let this aggregate be denoted

$$\Delta(Z)$$

and let

$$\Sigma(Z)$$

represent total supply, similarly obtained.

Rationing of the product will be assumed to be in accordance with the following simple, and extreme, mechanism:

If $\Delta(Z) < \Sigma(Z)$, the fraction $\Delta(Z)/\Sigma(Z)$ of suppliers can sell their product at the price (q) : the remainder lose their sales and receive nothing. The intermediate product is assumed to be non-durable. Similarly, if $\Sigma(Z) < \Delta(Z)$, then $\Sigma(Z)/\Delta(Z)$ have their demands met exactly and the remainder receive nothing. Though very severe, this rationing scheme captures the idea of temporary shortages and loss of sales. More moderate rationing methods probably temper the sharpness of our results but not the broader qualitative features of the model.¹²

We suppose that the capital installed in the two production modes (upstream and downstream) is fixed and not transferable between them. The total quantities of capital are denoted as

$$\bar{K}^1 \text{ and } \bar{K}^2,$$

for the production of the intermediate product and the final product respectively.

To avoid technical problems (like the indivisability of firms) and because the size of firms is indeterminate as a result of our constant-returns-to-scale assumption, we identify 'firms' with infinitesimal quantities of capital. Upstream firms are infinitesimals dK^u , where K^u is that part of $\bar{K}^1$ in the hands of upstream firms. Similarly, dK^d is the capital of a downstream firm. Integrated firms are to be thought of as combinations of infinitesimal units of capital of both types, linked together in the following way. Non-integrated firms must express their desired sales and purchases on a market where rationing is a possibility. Integrated firms, however, can pass the output of their upstream divisions directly to their downstream branches without running the risk of being rationed. The proportions of the two kinds of capital in an integrated firm is one of the variables determined by the model.

In any situation, the state of the industry is described by the organisational structure of the market – how much of each type of capital is owned by each class of firms – and by the actions of each of these firms. This determines the expected profits per unit of capital in the upstream and downstream categories and the expected profits accruing to the mixture of capital in the integrated firms. If all categories of firms are active, we shall say that the system is in equilibrium: (a) if the expected profits per unit of capital of the disintegrated firms (weighted by the proportions of the capital-types in integrated firms) equals the expected profits accruing to the corresponding quantity of capital in integrated firms; and (b) if no different proportions of capital-types for integrated firms (with the same yield in the non-integrated sector), would lead to higher expected profits. For states in which one or two of the categories of firms are empty, equilibrium requires, in addition, that the capital used in this mode would not earn a greater return than it does in the ongoing situation.

(2) BEHAVIOUR OF FIRMS

Each firm must decide how much of the intermediate product to demand or supply. In addition, upstream and integrated firms must choose an amount of labour. After making these decisions, the amount of exogenous demand becomes known, and rationing takes place as described above. If upstream or downstream firms are rationed, sales or output are zero respectively. Integrated firms, if rationed, carry out their downstream production using only their internally-generated supply of intermediate product. If the integrated firm was participating as a seller in the intermediate product market, the unsold quantity is added to his inputs.¹³

From the structure of the rationing process and of production, it is clear that the net demands per unit of capital by upstream and downstream firms are fixed at the levels corresponding to the efficient operation of their activities. There is no advantage in distorting desired sales or purchases since, by assumption, the probability of being rationed is independent of the firm's actions. Any divergence between stated desires and the efficient level can only decrease profits in the event that rationing does not affect the firm.

In the case of the integrated firm, however, the optimal level of excess demand for the intermediate product for it to present to the market is not independent of the choice of the optimal-integration proportions. The optimal-integration proportions are those which

maximise the excess of the total return to the capital of the integrated firm over the return that this capital would have earned if it had been used in non-integrated production processes. Let r^u and r^d be the expected returns per unit of K^u and K^d , in upstream and downstream firms respectively. Thus the optimal-integration proportions are determined by seeking the maximum expected profit for an integrated firm subject to;

$$r^u K^1 + r^d K^2 = 1. \quad (4)$$

Let the K^1 and K^2 satisfying (4) be fixed temporarily. Efficient operation of the firm requires that:

$$L = \frac{K^1 a'_L}{a'_K} \quad (5)$$

The level of net demand (z) on the intermediate product market that would lead to efficient production in the downstream stage is given by

$$z = \frac{K^2 b'_z}{b'_K} - \frac{K^1}{a'_K}, \quad (6)$$

and the corresponding level of final product is

$$v = \frac{K^2}{b'_K}. \quad (7)$$

If purchases of the intermediate product exceed this level of z , no additional (v) is produced. For lower levels of purchases (or higher levels of sales) the final output is given by

$$\begin{aligned} v &= \frac{K^1}{a'_K b'_z} + \frac{z}{b'_z} & \text{if } z > \frac{-K^1}{a'_K} \\ &= 0 & \text{if } z \leq \frac{-K^1}{a'_K} \end{aligned} \quad (8)$$

The opportunity loci for (v) and (z), at various integration proportions satisfying (4) are depicted in Figure 6.1.

It follows from the above, that the slope of the locus of efficient points is

$$\frac{dv}{dz} = \frac{\frac{dv}{dK^2}}{\frac{dz}{dK^2}} = \frac{1}{b'_z + \frac{r^d b'_K}{r^u a'_K}} \quad (9)$$

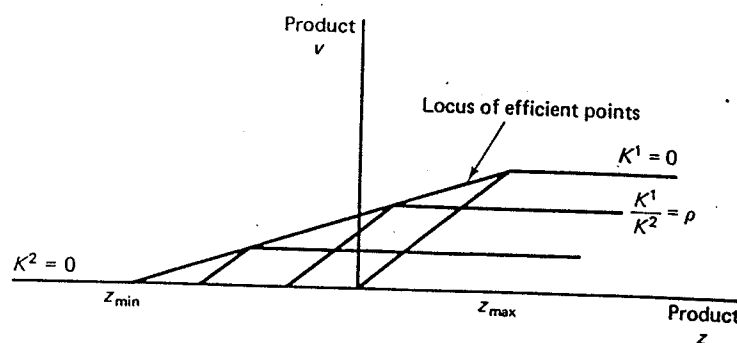


FIG. 6.1

and the maximal and minimal levels of (z) consistent with efficient-firm operation are

$$z_{\max} = \frac{b'_z}{r^d b'_K}$$

$$z_{\min} = \frac{-1}{r^u a'_K} \quad (10)$$

At these extreme points, the integrated firm is acting as if it were a purely downstream or a purely upstream produce, though with a different technology.

We shall now argue that, given the assumptions made thus far, the optimal configuration for an integrated firm is such that it operates efficiently, without any participation in the intermediate-product market. It will choose K^1 and K^2 so that the upstream division produces precisely the amount of intermediate product required downstream. Any other choice of K^1/K^2 requires *either* some inefficiency *or* some probability of the rationing of sales or purchases, with consequent economic losses.

Intuitively, the reason for this result is that the inferior input requirements of the integrated firm are independent of the proportions of capital it chooses to employ. Therefore, once it has decided to adopt the integrated form, the only remaining consideration is the minimisation of rationing possibilities.

A rigorous demonstration of this proposition is given in the Appendix. For the subsequent development of this chapter it suffices to note that the efficient mixture of the types of capital is given by

$$\rho^* = \frac{K^1}{K^2} = \frac{a'_K b'_z}{b'_K}.$$

We shall also refer to this ratio as the *balanced proportions* of integration.

III EQUILIBRIUM

(1) PROFIT AS A FUNCTION OF INDUSTRY STRUCTURE

In order to determine whether a particular structure of the firms in this industry is in equilibrium, we must compute the expected-profit levels for each type of firm, as it depends on the parameters of the equilibrium. Let

$$k^A(Z) = \min \left(\frac{\Delta(Z)}{\Sigma(Z)}, 1 \right), \text{ and}$$

$$k^B(Z) = \min \left(\frac{\Delta(Z)}{\Sigma(Z)}, 1 \right)$$

be the proportions of suppliers and demanders of intermediate product who are able to realise their desired quantities. Let E^A and E^B be the means (averages) of these two random variables, namely, the expected quantities realised by participants in this market. These, it has been argued above are, respectively, the upstream and downstream non-integrated firms.

Profits of the upstream firms per unit of K^u are

$$E^A \frac{q}{a_K} - \frac{a_L}{a_K} = r^u. \quad (11)$$

Profits of downstream firms per unit of K^d are

$$E^B \left(\frac{p - qb_y}{b_K} \right) = r^d. \quad (12)$$

Profits of integrated firms will be independent of rationing since they chose balanced proportions. If we consider such an integrated firm, with capital K^1 and K^2 satisfying

$$K^1 = \frac{a'_K b'_z}{b'_K} K^2, \quad (13)$$

the excess of revenues from the sale of final product over labour costs is

$$K^2 \left(\frac{p - a'_L b'_z}{b'_K} \right). \quad (14)$$

In any equilibrium in which both integrated and non-integrated firms are operating,

$$\left(\frac{p - a'_L b'_z}{b'_K} \right) = r^d + \frac{a'_K b'_z}{b'_K} r^u.$$

or, using (11) and (12)

$$E^A \left(\rho^* \frac{q}{a_K} \right) + E^B \left(\frac{p - q b_y}{b_K} \right) = \frac{p}{b'_K} + \left(\frac{a_L}{a_K} - \frac{a'_L}{a'_K} \right) \rho^* \quad (15)$$

This defines the class of rationing frequencies consistent with mixed equilibria of this type. The right-hand side of (15) is constant, whereas the left-hand side will vary with the extent of vertical integration in the industry, because of the induced changes in $k^A(Z)$ and $k^B(Z)$ and consequently in E^A and E^B . If the left-hand side of (15) is greater than the right-hand side, then integrated firms will tend to dissolve into separate enterprises in order to take advantage of the superior technologies available to these firms. Likewise, the reverse inequality sets up forces favouring integration as firms attempt to avoid the stochastic rationing on both sides of the market.

In any situation that is potentially an equilibrium one, the amount of capital in the three types of firms is given by K^u (upstream), K^d (downstream) and $(\bar{K}^u - K^u, \bar{K}^d - K^d)$ (integrated), where

$$\frac{\bar{K}^u - K^u}{\bar{K}^d - K^d} = \rho^*.$$

The expected values of the rationing-parameters depend on the degree of integration because the total demand and supply for intermediate product varies with the size of the upstream and downstream parts of this industry. The supply of intermediate product is given by

$$\sigma = \frac{K^u}{a_K}, \quad (16)$$

and the demand by downstream firms is

$$\delta = \frac{b_y K^d}{b_k}, \quad (17)$$

in accordance with the discussion at the beginning of Section II(2).

Total demand and supply are defined by

$$\begin{aligned} \Sigma(Z) &= \sigma + S - Z_- \\ \Delta(Z) &= \delta + D + Z_+. \end{aligned} \quad (18)$$

Using (18), the changes in rationing frequencies for non-integrated firms with respect to changing conditions in the intermediate product market are given by

$$dE^A = \int_{\{Z|k^A < 1\}} \frac{\Sigma(Z)d\delta - \Delta(Z)d\sigma}{[\Delta(Z)]^2} f(Z) dZ \quad (19)$$

$$dE^B = \int_{\{Z|k^B < 1\}} \frac{\Delta(Z)d\sigma - \Sigma(Z)d\delta}{[\Delta(Z)]^2} f(Z) dZ$$

when integration takes place in the balanced proportions

$$\frac{dK^u}{dK^d} = \rho^* \quad (20)$$

and hence

$$d_\sigma = \rho^* \frac{b_k d\delta}{a_k b_y} = \frac{a'_k b'_k b'_z}{a_k b'_k b_y} d\delta. \quad (21)$$

Substituting in (19)

$$\begin{aligned} dE^A &= d\delta \int_{\{Z|k^A < 1\}} \left[\frac{1}{\Sigma(Z)} - \frac{\Delta(Z)}{[\Sigma(Z)]^2} \frac{a'_k b'_k b'_z}{a_k b'_k b_y} \right] f(Z) dZ \\ dE^B &= d\delta \int_{\{Z|k^B < 1\}} \left[\frac{a'_k b'_k b'_z}{a_k b'_k b_y} \cdot \frac{1}{\Delta(Z)} - \frac{\Sigma(Z)}{[\Delta(Z)]^2} \right] f(Z) dz \end{aligned} \quad (22)$$

Consider the two terms in the integrand of the expression for dE^A . Whenever Z is in the range of integration,

$$\frac{1}{\sigma + S + Z_-} > \frac{\delta + D + Z_+}{(\sigma + S + Z_-)^2} \quad (23)$$

As the industry becomes more integrated, $d\sigma$ and $d\delta$ are negative. Hence

$$dE^A < d\delta \int_{\{Z|k^A < 1\}} \frac{\Delta(Z)}{[\Sigma(Z)]^2} \left(1 - \rho^* \frac{b_K}{a_K b_Y}\right) f(Z) dZ \quad (24)$$

and

$$dE^B < d\delta \int_{\{Z|k^B < 1\}} \frac{\Sigma(Z)}{[\Delta(Z)]^2} \left(\rho^* \frac{b_K}{a_K b_Y} - 1\right) f(Z) dZ. \quad (25)$$

Let $M = \max(\sigma + S, \delta + D)$ and $N = 1 - \rho^* b_K/a_K b_Y$.

The parameter (N) is central to the analysis. When $N = 0$, the technological disadvantage faced by integrated firms is neutral in the following sense. When firms integrate in balanced proportions, the decreased demand for intermediate product by previously downstream firms exactly offsets the decreased supply due to there being a smaller upstream sector. When $N > 0$, integration absorbs more of the demand than it does of the supply. Both decrease, but the net effect on excess demand is negative. Therefore the sign of (N) is a key benchmark for ascertaining the effects of integration on rationing frequencies. At $N = 0$, we can be sure that integration causes rationing more frequently on both sides of the market; in general, the answer will depend on the sign of N and on the distribution of Z , as developed below.

If $N > 0$, we have that as the degree of integration increases,

$$dE^B < -d\delta \frac{N}{M} \text{Prob}(k^B < 1)$$

$$dE^A < d\delta N \int \frac{\Delta(Z)}{[\Sigma(Z)]^2} f(Z) dZ. \quad (26)$$

Let $\Delta_{\min} = \delta + D + \min Z_+$

$$\Sigma_{\min} = \sigma + S - \min Z_-,$$

where the minimum of Z_+ and Z_- is taken over all Z that occur with positive probability under the distribution F .

We have that

$$dE^A < -d\delta N \frac{\Delta_{\min}}{\Sigma_{\min}^2} \text{Prob}(k^A < 1) \quad (27)$$

using the facts the $d\delta < 0$ and $N > 0$.

Therefore an upper bound on the total derivative with respect to increasing integration of the left-hand side of (15) is given by

$$\begin{aligned} & - \left(\frac{p - qb_y}{Mb_K} \frac{\Delta_{\min} \rho^* q}{\Sigma_{\min}^2 a_K} \right) Nd\delta \text{Prob}(k^B < 1) \\ & + Nd\delta \frac{\rho^* q}{a_K} \frac{\Delta_{\min}}{\Sigma_{\min}^2} \end{aligned} \quad (28)$$

Similarly, if $N < 0$, we calculate this upper bound as

$$\begin{aligned} & - \left(\frac{p - qb_y}{b_K} \frac{\Sigma_{\max}}{\Delta_{\max}^2} + \frac{\rho^* q}{a_K M} \right) Nd\delta \text{Prob}(k^B < 1) \\ & + Nd\delta \frac{\rho^* q}{Ma_K} \end{aligned} \quad (29)$$

Expressions (28) and (29) are useful for deriving the qualitative properties of the equilibria in the degree of integration in the industry. Recall that the equality (15) describes the co-existence of integrated and non-integrated firms in equilibrium. If the left-hand side of (15) is decreasing with integration, such an equilibrium is unstable and the qualitative characteristics of the industry are as asserted in Section I. Note that, when $N = 0$, this basic instability result is valid because (28) and (29) – which are upper bounds on this derivative – are both zero. This accords well with intuition. The case $N = 0$ implies the $d\delta = d\sigma$, and hence k^A and k^B both decrease for each value of Z because subtracting equal small quantities from each of two positive numbers with a ratio below one causes this ratio to decrease. Therefore both E^A and E^B will decrease in this situation, and both types of non-integrated firms become less profitable.

Consider $N > 0$. By (21)

$$0 > d\sigma > d\delta \quad (30)$$

as integration increases.

Let $\bar{Z}$ be the value of the stochastic exogenous excess demand that satisfies

$$\delta + D + \bar{Z}_+ = \sigma + S - \bar{Z}_- \quad (31)$$

$$\text{Since } Z = Z_+ + Z_-,$$

$$\bar{Z} = \sigma + S - (\delta + D) \quad (32)$$

when (30) holds

$$d\bar{Z} = d\sigma - d\delta > 0$$

and hence, since

$$\text{Prob}(k^B < 1) = \int_{\bar{Z}}^{\infty} f(Z) dZ \quad (33)$$

$\text{Prob}(k^B < 1)$ decreases with increasing integration in this industry. If $N < 0$, the opposite result obtains.

Referring to (28) and (29), this implies that both upper bounds on the slope of the weighted disintegrated profits function decrease with the extent of integration. This is the principal result of this section. Sufficient conditions for the slope of the weighted disintegrated profits function to be everywhere negative can be obtained from (28) and (29) as

$$\text{Prob}(k^B < 1) < \frac{\frac{\rho^* q}{a_K} \frac{\Delta_{\min}}{\Sigma_{\min}^2}}{\frac{p - qb_y}{Mb_K} + \frac{\rho^* q_{\min}}{a_K \Sigma_{\min}^2}} \quad (34)$$

when $N > 0$, and

$$\text{Prob}(k^B < 1) > \frac{\frac{\rho^*}{Ma_K} q}{\frac{p - qb_y}{b_K} \frac{\Sigma_{\max}}{\Delta_{\max}^2} + \frac{\rho^* q}{Ma_K}} \quad (35)$$

when $N < 0$.

Although the possibility of a rising portion of the weighted disintegrated profits function may lead to stable equilibrium at intermediate levels of integration, the fact that this is impossible when $N = 0$ and unlikely when $|N|$ is small makes the study of this unstable case useful as a benchmark.

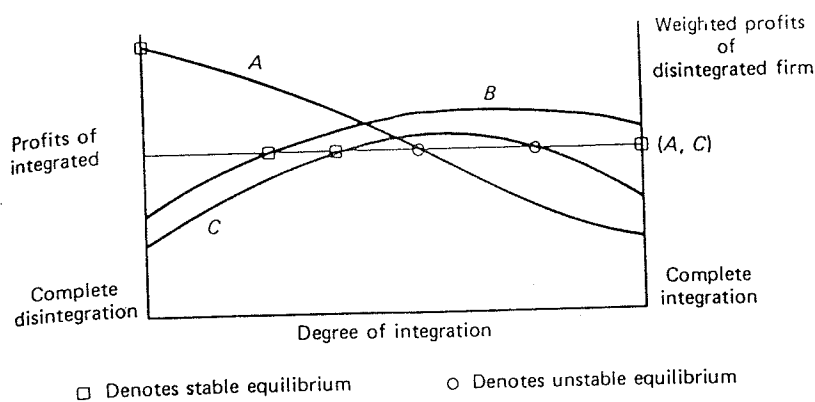


FIG. 6.2

(2) THE STRUCTURE OF EQUILIBRIA

The above analysis indicates how the structure of equilibria in this industry relates to the change in technology when integration takes place. Some possibilities are shown in Figure 6.2.

$$\text{Let } K_{\max}^1 = \min(\bar{K}^u, \rho^* \bar{K}^d).$$

We may then regard the degree of integration as $K^1/(K_{\max}^1) \in [0,1]$. Suppose, as Stigler (1951) and a variety of empirical studies suggest, that new industries are 'born' vertically integrated, perhaps because they use intermediate products that are specially tailored for their technology and for which alternative suppliers are not yet reliable producers. Only cases A and C are possible if complete integration is an equilibrium. If the industry grows proportionately by expanding $\bar{K}^u$ and $\bar{K}^d$ at the same rate, at which D and S are growing while $f(Z)$ remains the same, it is clear that E^A and E^B are increasing over time, viewed as functions of $K^1/K_{\max}^1$. Thus, weighted expected and disintegrated profits are rising for each fixed value of $K^1/K_{\max}^1$. At the point at which complete integration ceases to be an equilibrium, we have situation A^* or C^* as shown in Figure 6.3.

In either instance, the system experiences a qualitative discontinuity, arriving either at complete disintegration or at a substantially-reduced level of integration, according to whether the profit functions are of the form A or C respectively. This economic 'catastrophe'¹⁴ seems worthy of empirical testing. The theoretical implication of a reduction of integration as the market grows is identical with Stigler's proposition

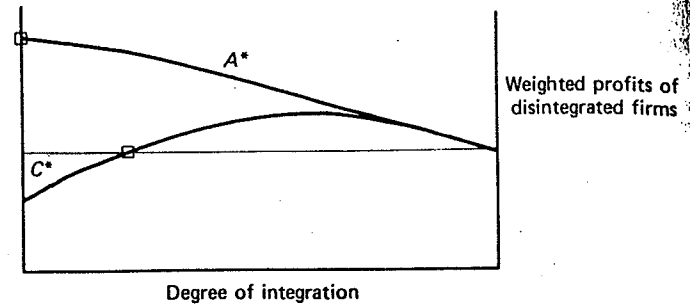


FIG. 6.3

(which is based on assumptions about returns to scale). But the dynamic implications of this model should allow one to discriminate between these theories.

A similar phenomenon is predicted when an initially non-integrated industry experiences increasing fluctuations in the intermediate product market. A discontinuous shift to complete vertical integration is indicated in this case.

IV WELFARE CONSIDERATIONS

The model we have presented is fraught with externalities. Each organisational decision to make an incremental in the quantity of capital exerts a real effect on the profitability of every firm in the disintegrated state. It is natural to study the welfare implications of this model. What is the optimal degree of vertical integration? What is the social cost of the price rigidity that generated the incentive for integration? Should integration and divestiture be encouraged or discouraged by public policy?

To approach these issues, we must first calculate the net social contribution made by the sector under examination. The value of the labour input is a non-stochastic quantity given by

$$\begin{aligned}
 L &= \frac{a_L}{a_K} (\bar{K}^u - K^1) + \frac{a'_L}{a'_K} K^1 \\
 &= \frac{a_L}{a_K} \bar{K}^u - K^1 \left(\frac{a_L}{a_K} - \frac{a'_L}{a'_K} \right). \quad (36)
 \end{aligned}$$

Output of the final product is stochastic, however, due to the impact of the rationing process on non-integrated firms. Expected output is given by

$$Ev = \frac{\bar{K}^d - K^2}{b_K} E^B + \frac{K^2}{b'_K} = \frac{\bar{K}^d E^B}{b_K} - \frac{K^1}{\rho^*} \left(\frac{E^B}{b_K} - \frac{1}{b'_K} \right). \quad (37)$$

To evaluate the net contribution of the industry since its participation in the market for the intermediate product, we must consider several cases. If the net demand of this industry, $\delta - \sigma$, has the same sign as the exogenous net demand $D - S + Z$. The industry is of no social value in this respect, for it does not satisfy any needs in the economy; nor does it use resources that would otherwise have been employed elsewhere. If $\delta - \sigma$ is positive and $D - S + Z$ is negative, the net social contribution is also zero, since this industry is utilising resources that would otherwise have been wasted. On an overall basis there is a gain, but it will appear in the form of an increase in the amount of final product produced.

However, if $\delta - \sigma$ is negative and $D - S + Z$ is positive, the industry is fulfilling a need elsewhere in the economy. The extent to which it is adding to total social benefits may be limited by the size of the exogenous demand to be filled, if this is smaller than the supply forthcoming. Thus the expected net contribution in the intermediate product market is:

$$\begin{aligned} & E \min (\sigma - \delta, D - S + Z) \quad \text{if } \sigma - \delta > 0, \\ & \quad \quad \quad (Z|D-S+Z>0) \\ & 0 \quad \quad \quad \text{if } \sigma - \delta < 0. \end{aligned} \quad (38)$$

The total expected social value of this sector [$W(K^1)$] can therefore be written as

$$\begin{aligned} W(K^1) = W_+(K^1) &= \frac{p\bar{K}^d E^B}{b_K} - \frac{a_L \bar{L}^U}{a_K} + K^1 \left[\frac{p}{\rho^*} \left(\frac{1}{b'_K} - \frac{E^B}{b_K} \right) \right. \\ & \quad \left. + \left(\frac{a_L}{a_K} - \frac{a'_L}{a'_K} \right) \right], \end{aligned} \quad (39)$$

if $\delta - \sigma > 0$;

$$\begin{aligned} \text{or} \quad W(K^1) &= W_+(K^1) + qE \min (\sigma - \delta, D - S + Z), \\ & \quad \quad \quad (Z|D-S+Z>0) \\ & \quad \quad \quad \text{if } \delta - \sigma < 0. \end{aligned} \quad (40)$$

Here, we have valued each commodity at its *fixed* price, an admittedly imperfect approximation.

The evaluation of $W(\cdot)$ is in general quite complex. If equilibria are generated by a profit function for which only the extreme organisational forms represent equilibria, then the relevant consideration is the comparison of $W(0)$ and $W(K_{\max}^1)$. If the weighted non-integrated profit function is decreasing and both complete disintegration and complete integration are stable equilibria, we have:

$$\begin{aligned} E^A(0) \left(\frac{\rho^* q}{a_K} \right) + E^B(0) \left(\frac{(p - qb_y)}{b_K} \right) &\geq \frac{p}{b'_K} + \left(\frac{a_L}{a_K} - \frac{a'_L}{a'_K} \right) \rho^* \\ &\geq E^A(K_{\max}^1) \left(\frac{\rho^* q}{a_K} \right) + E^B(K_{\max}^1) \left(\frac{(p - qb_y)}{b_K} \right). \end{aligned} \quad (41)$$

Here, we have written E^A and E^B as functions of the degree of integration.

It will again be simplest to treat the case in which $N = 0$ so that $\sigma - \delta$ is a constant with respect to the degree of integration. Hence,

$$\begin{aligned} W(K_{\max}^1) - W(0) &= \frac{p\bar{K}^d}{b_K} \left(E^B(K_{\max}^1) - E^B(0) \right) \\ &\quad + K_{\max}^1 \left[\frac{p}{\rho^*} \left(\frac{1}{b'_K} - \frac{E^B(K_{\max}^1)}{b_K} \right) \right. \\ &\quad \left. + \left(\frac{a_L}{a_K} - \frac{a'_L}{a'_K} \right) \right] \end{aligned} \quad (42)$$

Suppose that the industry is in stable equilibrium with complete integration and that the environment of the industry is changing so that weighted disintegrated profits are rising relative to integrated profits. We can ask whether the switch to the disintegrated mode occurs too early or too late in this process. That is, should integration be encouraged or discouraged in the course of the process? At the switch point, the last of the inequalities (41) is an equality.

Solving this for $(a_L/a_K - a'_L/a'_K)$, substituting the result in (42) and using the fact that $N = 0$, we have:

$$\begin{aligned}
W(K_{\max}^1) - W(0) &= \frac{pE^B(K_{\max}^1)}{b_K} \left(\bar{K}^d - \frac{K_{\max}^1}{\rho^*} \right) \\
&+ \frac{qK_{\max}^1}{a_K} \left(E^A(K_{\max}^1) - E^B(K_{\max}^1) \right) \\
&+ \frac{p}{b_K} \left(\frac{K_{\max}^1}{\rho^*} E^B(K_{\max}^1) - \bar{K}^d E^B(0) \right). \quad (43)
\end{aligned}$$

We can divide the possibilities into two cases:

Case I: $\rho^* \bar{K}^d = K_{\max}^1$;

Case II: $\rho^* \bar{K}^d > K_{\max}^1$; $= \bar{K}^u$.

In Case I, the change in welfare reduces to:

$$\begin{aligned}
W(K_{\max}^1) - W(0) &= \bar{K}^d \left[\frac{p}{b_K} \left(E^B(K_{\max}^1) - E^B(0) \right) \right. \\
&\left. + \frac{\rho^* q}{a_K} \left(E^A(K_{\max}^1) - E^B(K_{\max}^1) \right) \right] \quad (44)
\end{aligned}$$

In Case II we have:

$$\begin{aligned}
W(K_{\max}^1) - W(0) &= \frac{p}{b_K} \left(\frac{\bar{K}^u}{\rho^*} E^B(\bar{K}^u) - \bar{K}^d E^B(0) \right) \\
&+ \bar{K}^u \left[\frac{q}{a_K} \left(E^A(\bar{K}^u) - E^B(\bar{K}^u) \right) \right] \\
&+ \frac{pE^B(\bar{K}^u)}{b_K} \left(\bar{K}^d - \frac{\bar{K}^u}{\rho^*} \right). \quad (45)
\end{aligned}$$

The first term in each of the expressions (44) and (45) is negative when $N = 0$, because E^B is a decreasing function. The implication of this is that divestiture will tend to occur *too late*, from a social point of view. This is so in the sense that complete disintegration will be the superior organisational form when complete integration could still persist as a stable equilibrium.

The sign of the second term in each is ambiguous, indicating that the first effect will be mitigated if sellers are rationed less heavily than buyers, and will be reinforced in the opposite instance.

Finally, the last term in (45) is positive; there is a force operating in favour of postponing divestiture if the downstream sector is larger than the upstream one.

Turning now to the instance in which integration is about to take place in the industry, we have an exact equality in the left-hand relation of (41). Following a procedure similar to that above, one obtains the result that in Case I,

$$W(K_{\max}^1) - W(0) = \frac{\bar{K}^d \rho^* q}{a_K} \left(E^A(0) - E^B(0) \right); \quad (46)$$

and in Case II,

$$W(K_{\max}^1) - W(0) = \frac{K_{\max}^1 q}{a_K} \left(E^A(0) - E^B(0) \right) + \left(\bar{K}^d - \frac{K_{\max}^1}{\rho^*} \right) \left(E^B(K_{\max}^1) - E^B(0) \right) \frac{p}{b_K}. \quad (47)$$

The first term in (46) and (47) is of ambiguous sign. It indicates that integration will occur too early from a social point of view if sellers are rationed more heavily than buyers in the disintegrated state, and conversely. The second term in (46) is unambiguously negative. This implies that integration will occur *too early*, other things being equal.

Comparing the results of the Section in the cases of integration and disintegration, we note that the socially-preferred equilibrium will be biased toward disintegration if sellers are rationed more severely than buyers; and that there are further forces favouring disintegration due to the differential rationing frequencies in the two potential equilibria.

Now consider the more general situation where $N \neq 0$, but still under the assumption that only complete integration and complete disintegration are potential stable equilibria. We must now add a term representing the change in the supply of the intermediate product to the exogenous market.

The change in welfare when integration gives way to disintegration is given by:

$$\begin{aligned}
W(K_{\max}^1) - W(0) = & \frac{pE^B(K_{\max}^1)}{b_K} \left(\bar{K}^d - \frac{K_{\max}^1}{\rho^*} \right) \\
& + p \left(\frac{K_{\max}^1}{\rho^* b'_K} - \frac{\bar{K}^d E^B(0)}{b_K} \right) \\
& + K_{\max}^1 \left[q \left(\frac{E^A(K_{\max}^1)}{a_K} - \frac{E^B(K_{\max}^1) b_y}{\rho^* b_K} \right) \right. \\
& \left. + \frac{p}{\rho^*} \left(\frac{E^B(K_{\max}^1)}{b_K} - \frac{1}{b'_K} \right) \right] + X, \quad (48)
\end{aligned}$$

where $X = q [E \min (\max (\sigma^{\max} - \delta^{\max}, 0), \max(D - S + Z), 0) - E \min (\max (\sigma^0 - \delta^0, 0), \max(D - S + Z), 0)]$.

Here, σ^0 , δ^0 , $\sigma^{\max}$, and $\delta^{\max}$ are respectively, the supplies and demands of this industry in its extreme organisation structures.

If integration is about to take place, we have

$$\begin{aligned}
W(K_{\max}^1) - W(0) = & \frac{pE^B(K_{\max}^1)}{b_K} \left(\bar{K}^d - \frac{K_{\max}^1}{\rho^*} \right) \\
& + p \left(\frac{K_{\max}^1}{\rho^* b'_K} - \frac{\bar{K}^d E^B(0)}{b_K} \right) \\
& + K_{\max}^1 \left[\frac{p}{\rho^*} \left(\frac{E^B(0)}{b_K} - \frac{1}{b'_K} \right) \right. \\
& \left. + \frac{qE^A(0)}{a_K} - \frac{qE^B(0) b_y}{\rho^* b_K} \right] + X. \quad (49)
\end{aligned}$$

Note that, by definition, $N > 0$ implies:

$$\rho^* < \frac{a_K b_y}{b_K} \quad \text{and,} \quad (50a)$$

$$\sigma^m - \delta^m > \sigma^0 - \delta^0 \quad (50b)$$

Therefore the effect of N having a positive value is seen in (48) and (49) in two ways: The next to the last term becomes smaller in each,

due to (50a); but $X > 0$, by (50b). Conversely, $N < 0$ also leads to two conflicting influences on the net change in social welfare. Therefore we can say that although $N \neq 0$ may modify the results above, it introduces no unambiguous systematic bias in the system.

In cases where changes in slope of the weighted disintegrated profits function induce equilibria other than with the extreme organisational forms, the planner must be able to compute the stable equilibrium to which the system will converge. Only then can he carry out the analysis. Having done so, however, the welfare considerations will parallel those above.

Finally, we should remark on the rigidity of prices which is the source of the incentives for vertical integration discussed in this chapter. We have assumed that the prices (p and q) which relate to products for which the demand is fluctuating over time cannot respond at all to these variations. In the analysis above, we assumed that they did not vary with the organisation of the industry as well. However, it is natural to suppose that the level of these prices *do* respond to the average excess demand in their respective markets over time. Therefore, when the degree of integration varies, prices will adjust to the new distribution of excess demands and this will induce changes in the social product attributable to this industry.

On the intermediate-product market, as we have seen, the distribution of excess demand shifts upward with increasing integration if $N < 0$, and falls if $N > 0$. Thus the consumers' surplus will increase with integration in the latter case, and this must be taken into account in deciding whether to permit such mergers.

In the market for final product, we may suppose that price will vary in response to shifts in the expected value of this output, demand conditions being treated as exogenous. From (37) we have the result that when integration replaces disintegration as the equilibrium, the change in output is given by:

$$\Delta(Ev) = \frac{\bar{K}^d}{b_K} \left(E^B(K_{\max}^1) - E^B(0) \right) - \frac{K_{\max}^1}{\rho^*} \left(\frac{E^B(K_{\max}^1)}{b_K} - \frac{1}{b'_K} \right). \quad (51)$$

In Case I ($\rho^* \bar{K}^d = K_{\max}^1$), this becomes:

$$\Delta(Ev) = \bar{K}^d \left(\frac{1}{b'_K} - \frac{E^B(0)}{b_K} \right); \quad (52)$$

and in Case II we have

$$\Delta(Ev) = \frac{\bar{K}^u}{\rho^* b'_K} - \frac{\bar{K}^d E^B(0)}{b_K} + \frac{E^B(K_{\max}^1)}{b_K} \left(\bar{K}^d - \frac{K_{\max}^1}{\rho^*} \right). \quad (53)$$

Therefore, consumers' surplus in the final product market will increase more with the switch to complete integration, *ceteris paribus*, if the rationing of buyers in the disintegrated state is severe. In Case II, we see that the second term induces a bias in favour of integration through the resulting fall in price. This would have to be offset against the opposing effect, as indicated in equations (45) and (47).

V REMARKS ON THE MODEL

The results in this chapter hinge crucially on two properties of the model; the non-participation of integrated firms in the market for the intermediate product, and the insensitivity of offers to buy and sell this good as the frequency of rationing changes. These are a consequence of the nature of the technology assumed and of the rationing rule. Below, we discuss the generalisation of each of these specifications in turn.

Suppose that, instead of a fixed-coefficients technology, we assume that production functions per (incremental) unit of capital are concave in the other input relevant to that stage of fabrication. Two differences from the analysis in Section II now appear. First, the set of efficient production plans for an integrated firm (given a fixed quantity of labour input) is non-linear in the composition of capital of the firm. Second, the optimal mode of operations given the proportions of integration is not determined independently of the extent of rationing.

Nevertheless, rationing affects an integrated firm adversely should it choose to enter in the intermediate product market on either side. Hence, the optimal action will be constant at $z = 0$ throughout a non-degenerate range of values of p and q , just as in the fixed-coefficient case. Neo-classical production functions do lose the property that no other form of integration can ever be optimal, as a consequence of the non-linearity mentioned above. However, if prices are in the range

mentioned, this range will be large when shortages are relatively severe. Our analysis, which depends only on the balanced nature of integrated firms, will go through essentially unchanged.

The robustness of our results to assumptions concerning the rationing rule is on less-certain ground. The problem is that if the extent to which any firm is rationed is responsive to its own actions, then firms will not have the incentive to transmit their technologically-optimal quantities to the market as desired transactions. This complicates the model considerably. But what is worse is that it may tend to destabilise the stochastic equilibrium with a fixed configuration of firms in the industry, rendering an analysis of the equilibrium industry-structure meaningless. In the extreme case of proportional rationing, which is the polar opposite of the allocation mechanism we have used, it is in the interest of downstream firms to exaggerate their demands for the intermediate product, balancing the expected gain in productive inputs received against occasional receipts of excessive quantities. An even more troublesome phenomenon arises with respect to sellers of the intermediate product. They have a natural incentive to promise deliveries in excess of production under any rationing scheme other than the one treated. The necessary legal structure for adjudicating contract violations would then have to be specified before any definitive analysis of their problem could be made; and this would lead us hopelessly far afield.

VI CONCLUSION

This chapter has attempted to study vertical integration in response to imperfect price flexibility. The model used is shown to possess an inherent instability for equilibria in which the industry is only partially integrated. The extremes of market structure are generally the only stable equilibria.

In evaluating whether mergers or divestitures desired by firms are socially advantageous, this property of the nature of potential equilibria was utilised. When divestiture is desired in a completely-integrated situation there are essentially three effects to consider. Vertical disintegration should be discouraged by public policy (1) if the difference in the rationing frequency for sellers will not be greatly improved in the completely disintegrated equilibrium, (2) if buyers are being rationed more heavily than sellers, or (3) if the downstream sector is large relative to the upstream. When integration is desired,

it should be discouraged if sellers are rationed more heavily than buyers in the existing equilibrium, or if the frequency of rationing will increase markedly after integration becomes complete.

APPENDIX

In this Appendix, we demonstrate that the integrated firms will choose to constitute themselves so that they do not participate in the intermediate product market at all. This involves comparisons of profitability at $Z = 0$, where the firm is insulated from rationing, from rationing, with $Z \neq 0$, in which case rationing would affect expected profits. Let

$$k^A(Z) = \min \left(\frac{\Sigma(Z)}{\Delta(Z)}, 1 \right) \text{ and}$$

$$k^B(Z) = \min \left(\frac{\Delta(Z)}{\Sigma(Z)}, 1 \right) \quad (\text{A.1})$$

and let E^A and E^B be the expected values of these random variables, respectively.

For K^1 and K^2 fixed, and any feasible chosen level of v and z , the firm's expected realised net outputs are given by

$$\begin{aligned} \bar{v} &= vE^B + v^0(1 - E^B) \\ \bar{z} &= zE^B \end{aligned} \quad (\text{A.2})$$

if $z > 0$, and

$$\begin{aligned} \bar{v} &= vE^A + v^0(1 - E^A) \\ \bar{z} &= zE^A \end{aligned} \quad (\text{A.3})$$

if $z > 0$, where v^0 is the net output of final product associated with $z = 0$ and the specified choices of K^1 and K^2 .

For any fixed K^1 and K^2 , the optimal quantities to register on the product markets, v and z , will be at the efficient level given by (6) and (7) due to the nature of the rationing process. Expected profits, therefore, are

$$p \bar{v} - q \bar{z} \quad (\text{A.4})$$

where $\bar{v}$ and $\bar{z}$ specified in (A.2) or (A.3).

The following diagram depicts the equivalent profit locus ($\bar{v}$, $\bar{z}$) for various integration proportions.

The optimal integration proportions are those which maximise expected profits on the equivalent profits locus. The integration proportions giving rise to $z = 0$ given by (8) as

$$\frac{K^1}{K^2} = \frac{a'_K b'_z}{b'_K} \equiv \rho^*. \quad (\text{A.5})$$

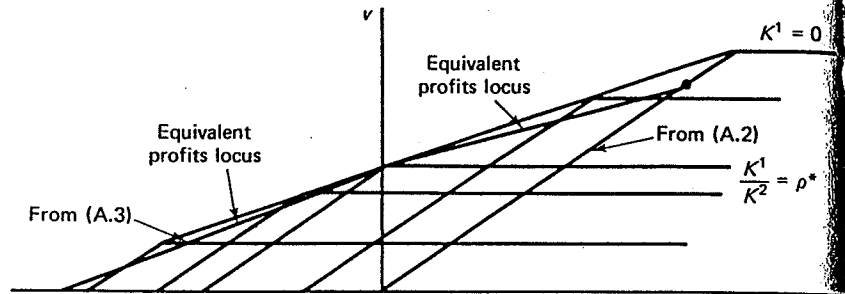


FIG. A.1

The ratio ρ^* will be called *balanced proportions* of integration. For K^1/K^2 above the balanced level, the equivalent profit locus has a slope of

$$\frac{1}{E^A b'_z} \left(\frac{1}{1 + \frac{r^d}{r^u \rho^*}} \right) \quad (\text{A.6})$$

since $z < 0$ for such firms and therefore $v^0 = v^{15}$.

For $K^1/K^2 < \rho^*$, the equivalent profit locus has a slope of

$$\frac{1}{E^B b'_z} \left(\frac{E^B \left(\frac{r^u \rho^*}{r^d} \right) - (1 - E^B)}{\frac{r^u \rho^*}{r^d} + 1} \right) \quad (\text{A.7})$$

since $z > 0$ and $v^0 = \frac{1 - r^d K^2}{r^u a'_K b'_z}$.

It is clear that if there is never any rationing, so that $E^A = E^B = 1$, the equivalent profit locus is linear, i.e. (A.6) and (A.7) are equal. As the former is decreasing in E^A and the latter is increasing in E^B , the equivalent profit locus has the general shape as indicated in Figure 6.2.

Suppose that K^1 and K^2 are varied in accordance with (4), from

$$\frac{K^1}{K^2} = \rho^*$$

to

$$\frac{K^1}{K^2} = 0$$

so that $z > 0$. We see that the change in the profit per unit change in K^2 is then given by

$$\begin{aligned} & \frac{1}{b'_K} \left[p \left(E^B \left(1 + \frac{r^d}{r^u \rho^*} \right) - \frac{r^d}{r^u \rho^*} \right) \right. \\ & \left. - q E^B \left(b'_z - \frac{r^d b'_K}{r^u a'_K} \right) + \frac{r^d a'_L b'_K}{r^u a'_K} \right] \end{aligned} \quad (\text{A.8})$$

Proceeding in the direction of a more-intensive downstream mode of operation as K^1/K^2 decreases towards ρ^* , the slope of the profit function is now given by

$$\frac{1}{b'_K} \left[p - q E^A \left(b'_z - \frac{r^d b'_K}{r^u a'_K} \right) + \frac{r^d a'_L b'_K}{r^u a'_K} \right].$$

The important thing to notice about (A.8) and (A.9) is that they are constants. Therefore, from the point of view of the (infinitesimal) firm, they are independent of the partitioning of K^1/K^2 within their respective ranges. If (A.8) is negative when (A.9) is positive, the optimal integration proportions are the balanced value, ρ^* . If their signs stand in any other configuration, we know that the optimal integration-proportions lie of one of the extremes – that is, the firm should behave as if it were actually a purely upstream or purely downstream operation. However, under these conditions, the firm would be better off to constitute itself as one of the pure varieties, thereby having access to a superior technology.

Summarising our results, we reach the conclusion that the only mode of operation ever observable for an integrated firm is the balanced one, in which it does not participate in the market for the

intermediate product. This will happen whenever $(A.8) < 0 < (A.9)$ which can be rewritten as linear inequalities relating the prices (p) and (q). The sharpness of these results is dependent crucially on the form of the production function, but the general qualitative property concerning the superiority of balanced integration for a broad class of prices and rationing frequencies is robust to the technological specification. Further discussion of this point has been postponed to Section V. For the remainder of the chapter we have assumed that $p, q \in E^A$ and E^B are always such that $(A.8) < 0 < (A.9)$. Thus, the issue of vertical integration is not an empty question.

NOTES

1. Theoretical studies of the feasibility and results of vertical integration in the presence of monopoly elements include Wu (1964), Edwards (1953), Machlup and Taber (1960), Vernon and Graham (1971), Schmalensee (1973) and Warren-Boulton (1974). In the public policy literature, Bork (1969), Mueller (1969) and Peltzman (1969) have made contributions to the understanding of the way in which various types of vertical control, among which outright merger is only one, are viewed under the law. Specific studies of industries with imperfectly competitive characteristics that have undergone vertical expansion are carried out in Adams and Dirlam (1964) and Dennison (1939), steel; Crandall (1968), automobile repair parts; Frankel (1953), de Chazeau and Kahn (1959), and McGee (1960), oil; and Peck (1960), aluminium, among others.
2. Adelman (1949), takes the view that integration is likely to increase output of the final product and reduce price. Stigler (1951) qualifies these remarks heavily.
3. See Arrow (1975) for the case in which vertical integration is used as a source of information by the downstream division. Other methods by which such information can be acquired or traded are discussed therein.
4. Williamson (1971) gives the most complete treatment of these issues. The basic problems were first pointed out by Coase (1937) and later explored in Malmgren (1961).
5. Irwin (1971) and Dayan (1972) have modelled this problem; less formal discussions are found throughout the business literature.
6. See Williamson (1971) and Allen (1971). In the case of the cement industry, the FTC study (US Federal Trade Commission, 1966) documents the market foreclosure thesis, though the industry structure has several aspects that complicate the picture – particularly increasing returns and high transportation costs that limit the extent of competition among both integrated and non-integrated firms in any region.
7. See Foley and Hellwig (1973), Leviatan (1971), Whitin (1974), Walters (1960), Rothschild (1969), Smith (1968) and Weitzman (1974) among others.

8. Representative of these models we have Barro and Grossman (1971), Benassy (1982), Clower (1965), Drèze (1975), Iwai (1972), Grandmont and Laroque (1976), Malinvaud (1976) and Younes (1975).
9. This point was first made by Malmgren (1961).
10. The validity of this assumption can certainly be called into question. We adopt it for the reason stated and because it corresponds in some sense to the usual condition of decreasing returns to scale. In the widely analysed case of cement-concrete mergers, industry opinion seems divided on the empirical facts. See Allen (1971, footnote 19) and Wilk (1968, footnote 47).
11. See Allen (1971), and the theoretical studies mentioned in Note 1.
12. Further discussion of this point is found in Section V.
13. This assumption is for concreteness only. It is also possible to treat the case in which the unsold intermediate input is not available for production.
14. Catastrophe theory, a branch of mathematics, was first used in economics by Zeeman (1974), and may fruitfully be applied to models of the structure of industries in more general contexts than that treated here.
15. A corresponding expression can be derived if it is assumed that unsold intermediate product is lost.

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