

PREEXISTING CONTRACTS
AND TEMPORARY GENERAL EQUILIBRIUM*

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10.1. Introduction

The object of this study is to consider a model of general economic equilibrium over time in which the markets for trading commodities are open at every date. This type of system is to be contrasted with that of the Arrow-Debreu model in which an equilibrium is reached at an initial point in time and the remainder of economic activity consists of carrying out the equilibrium plans formulated at that date. Two distinct approaches have been taken to the problem of modelling systems of this type. The first, introduced by Radner [13] and used subsequently by Hahn [6] and Kurz [10] in studies of transactions costs, is to ask whether there exists a set of prices on current markets and a sequence of point forecasts regarding prices on markets in the future such that, if these beliefs were held by everyone, the resulting course of the economy would see them fulfilled.

The second approach is that of viewing the economic system as a sequence of temporary equilibria. This idea goes back at least to Hicks [7]. It was introduced in a formal general equilibrium model of a monetary economy without futures markets by Grandmont [5]. Subsequently, Sondermann [4] treated a monetary economy with production in this framework and Green [3] studied a non-monetary economy with futures markets. In the temporary equilibrium approach, expectations are taken as data of the system, although they may depend on

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variables currently determined in an equilibrium. It is not required that individuals agree, though some degree of compatibility of expectations appears to be necessary to ensure the existence of an equilibrium (see ref. [3]).

The present paper is a continuation of ref. [3] in the following sense: if a temporary equilibrium in a previous period resulted in the exchange of futures contracts, then it is necessary to take into account the fact that the current period endowments will reflect these trades. This is not a problem if endowments, including preexisting contracts, remain in the consumption set. However, a sale of futures contracts in a previous equilibrium in excess of the endowment of that commodity may lead to the nonexistence of an equilibrium in the present period. In particular, the possibility of bankruptcy exists. This must be faced squarely if there is to be any hope that the economy can be modeled as a sequence of temporary equilibria when futures contracts are permitted.

10.2. Previous Work on Preexisting Contracts

There have been several previous attempts to incorporate preexisting contracts in a model of general equilibrium. In Arrow and Hahn [1] a single-period model is considered in which the endowments might contain negative amounts of some commodities due to debts incurred by the household in earlier periods¹. They prove that there exists a price system and a set of transfers (in units of account) among the agents such that all markets clear.

A second treatment of this topic is in the thesis of Grandmont [4]. Grandmont's primary interest is in an economy with money and in which money, or debt measured in units of account, is the only store of value. His model has two periods. In the first, endowments are non-negative and trade takes place only on spot markets. Debt, in units of account, can also be incurred, or claims to future units of account can be acquired. The price of obtaining one unit of account in the second period is determined in the equilibrium of the first period in the market

¹ Arrow and Hahn actually are more general on this point since they deal with a general consumption set instead of the non-negative orthant of the commodity space. Simplifying their model in this way will, however, facilitate comparison with what we shall do later.

for 'debts'. If an individual debtor is not solvent in period 2, because the value of his endowment is less than the amount of his debt, he is assigned the consumption of zero and his endowment is used to pay the debt to whatever extent this is possible. Naturally, if there is bankruptcy in the economy, the amount of debt repayments will be insufficient to cover the quantity owed. The repayments are made proportionately for all creditors. That is, every creditor is refunded the same proportion of what he is owed. The individuals in the economy know this rule in period 1, when they consider becoming creditors. They view loaning units of account as a risky prospect in the sense that they have subjective probability assessments over the fraction of debt that will be refunded.

Grandmont shows that there will be an equilibrium in each period: period 1, when endowments are non-negative and future bankruptcies are viewed with uncertainty; and period 2, when bankruptcies occur, debts are settled and economic activity ends.

The first introduction of the potential for bankruptcy in a general equilibrium model is due to Stigum [16], though his conditions for existence are not immediately comparable with those of the other authors.

Our model differs from each of these in several respects. As mentioned in the introduction, the primary objective is to be able to view the economic system as progressing from one temporary equilibrium to the next. Thus we will be considering a period in which there has been a past – preexisting contracts may be present – and there will be a future – plans are made viewing the future with subjective uncertainty. This differs from the Arrow–Hahn case in which there has been a past, but there will be no markets convening in the future. In the Grandmont model, period 1 has a future but no past, period 2 a past but no future. We shall attempt to synthesize these, so that the period in question can be viewed as a typical 'snapshot' of the economy.

One consequence of there being a future period is that individuals view their own potential bankruptcies, as well as those of others, with subjective uncertainty. If there is a past, the value of holding debts of others must be determined. Thus the two sides of the bankruptcy problem – anticipation and settlement – are simultaneously present in our system.

Our model has further differences with the Grandmont system

because contracts sold include all future commodities as well as those currently deliverable. Debt in our model will be represented by the selling of a futures contract for real goods instead of a claim to a future unit of account. Thus we will not be able to determine a price level as Grandmont does, but rather relative prices only will emerge. The system we propose, however, has many stores of value – every futures contract could serve as such. Typically, an individual may sell some contracts and buy others so that he can be classified as neither debtor nor creditor. The net value of his preexisting contracts is determined in the equilibrium. This gives rise to the possibility that the bankruptcy of one agent may cause others to be bankrupt if they are his creditors but in debt to others. This phenomenon cannot occur in the Grandmont system since all creditors will be solvent because their endowments are non-negative, even if no debtors repay any debts.

10.3. Codes of Conduct and Non-Economic Penalties in General Equilibrium Models

The Arrow-Debreu model presumes an institutional structure in which every agent has access to a system of markets that he can use to trade commodities. Since some of the commodities are not deliverable until a date beyond that at which trades are made, one typically speaks of these markets as involving futures contracts. An individual who buys such a contract believes that it will, in fact, be fulfilled: that is, that when the appointed date arrives, the exact amount of the agreed upon physical commodity will come into his possession. On the other side of the coin, therefore, the Arrow-Debreu model presupposes honesty: no one is allowed to sell a contract without having the required amount of the good available in his endowment (or production plan).

Thus the Arrow-Debreu model embodies an ethical code of conduct and a set of beliefs about the performance of the system, as well as an institutional framework. It should be noted that these are consistent in the sense that if everyone follows this code of conduct, the belief that all contracts will be fulfilled is justified. Further, the belief that the markets will only meet once is validated by the fact that even if it were possible to reopen them, no one would want to engage in trade at the equilibrium prices that would emerge.

However, certain generalizations of the Arrow-Debreu model may destroy this self-realization property. Knowledge that markets will reconvene will cause people to behave initially in such a way that they will desire to trade at future dates as well. In ref. [3] the existence problem was studied for such a system.

The result of an equilibrium in this model is a set of realized consumptions of current goods and an altered endowment distribution for the following period. In the following period endowments will be modified to reflect futures contracts traded. Thus this model tacitly assumes the Arrow-Debreu code of conduct and resulting beliefs about futures contracts. But, unless the endowments remain non-negative, these beliefs will prove unjustified. There may be no equilibrium because there are prices at which some individuals have endowments with negative value and therefore cannot choose any point in their consumption set (assumed to be the non-negative orthant). That is, the possibility of bankruptcy exists in this system even though every individual makes only those futures contracts that he can fulfill with subjective probability one. Some people hold subjective beliefs that are 'wrong' in the sense that the prices next period would have been deemed 'impossible' *ex ante*.²

The problem is that the institutional structure of sequential trading is incompatible (except under overly strong conditions on expectations) with the presumption that contracts are fulfilled with certainty. Thus a broader concept of the nature of economic contracts is required if we are to be able to view the system as generating a sequence of temporary equilibria in which the environment in each period is a result of past actions.

Since the possibility of bankruptcy cannot be avoided, a viable economic system must have rules governing this circumstance. The rule must be feasible to implement. That is, it must prescribe a feasible redistribution of commodities corresponding to any state of the economic system – including those states in which some bankruptcy is occurring.

Further, if one interprets consumption sets as describing consumption bundles that are the minimum required for sustenance of the individual, then some non-economic penalties for bankruptcy are required, for

² Strictly speaking, the actual prices are such that a neighborhood of them lies outside the convex hull of the support of the subjective distribution previously held.

otherwise an individual would not care about the extent of his bankruptcy – all bankruptcies would leave him with a consumption on the lower boundary of his feasible set. Problems concerning the determinateness of individual behavior would arise.

Thus, it seems that a natural consequence of the sequential markets setting is a need for explicit consideration of social rules of conduct and a penalty structure which are unnecessary in the Arrow–Debreu system and are usually thought to be outside the scope of economic analysis.

At this point a crucial choice in model-building arises: in the real world, the vast majority of contracts are bilateral in that the buyer and seller know each other's identity. For the purposes of the Arrow–Debreu model this was unimportant because, since all contracts are always fulfilled and transaction costs are absent, the identity of one's trading partner is economically irrelevant. If, however, we contemplate constructing a system in which these properties may fail, the identity of one's trading partner will matter and contracts with different people must be viewed differently; that is, they may have different prices in an equilibrium and must be viewed as distinct commodities.

One possibility would be to take explicit account of this phenomenon, making assumptions about the information each agent has regarding the portfolios of others. In an equilibrium of such a system, the prices of currently deliverable commodities will not differ among sellers, but the prices of futures contracts will.

Proving the existence of an equilibrium – and even stating its definition – in such a system raises several interesting and atypical questions. We hope to treat this in a separate paper.

For the present model, we shall assume that all trades are made with an abstract market. The contract obliges its seller to deliver the good, if he is solvent. A buyer, however, knows that some sellers may be bankrupt. The contract entitles the buyer to a proportion of its face value equal to the ratio of actual repayments to the total quantity of outstanding contracts for this commodity. This actual payout ratio will vary from commodity to commodity since the sellers of different futures contracts are, in general, different.

One way of interpreting this system is that every buyer is assigned, by the market, contracts of various sellers in proportion to his share of the total contracts held. If buyers were risk averse and had no information about the asset positions of any sellers, then this is the

allocation that they would choose in order to spread the risk evenly among themselves. Viewing our allocation rule in this way, it is apparent that this system (the use of an abstract market) is the polar opposite of the full information situation in which every buyer knows the complete asset position of every seller. However, since buyers' information is rarely perfect, and often very imperfect, this approach seems justified as a modeling technique by its symmetry and simplicity. It also seems to be a reasonable institutional rule because of its risk spreading properties under complete ignorance. We now proceed to a formal statement of the model and assumptions.

10.4. The Model

We will be studying the existence of a temporary equilibrium at a date that will be denoted by 1. There will be a future, denoted by period 2. Past economic activity enters the model through the date of period 1, as will be described shortly. There are ℓ_1 commodities at date 1 and ℓ_2 at date 2; we write $\ell \equiv \ell_1 + \ell_2$. Commodities are non-durable. A typical individual at date 1 is described as follows.

Naturally occurring endowment that accrues to him regardless of the actions of others and is known with no uncertainty is written

$$\omega = (\omega^1, \omega^2) \in R_+^\ell.$$

It is further assumed that $\omega^1 > 0$, $\omega^2 > 0$ and the aggregate is strictly positive³.

Preexisting contracts that reflect the economic activities of previous periods and are subject to default as we shall describe below are written

$$e = (e^1, e^2) \in R^l,$$

that is, in previous equilibria the individual may have traded contracts that are now due (e^1), or he may have made futures transactions involving periods still to come (e^2). Notice that no restriction is placed on the

³ We adopt the following conventions for vector inequalities:

$x \geq y$ implies $x_i \geq y_i$ for all i ;

$x > y$ implies $x_i \geq y_i$ for all i , and $x_i > y_i$ for some i ;

$x \gg y$ implies $x_i > y_i$ for all i .

sign of e . However, we will later use the fact that the sum of preexisting contracts over individuals must be zero.

Let us denote that $e = e_+ + e_-$, where $e_+ = \max(e_k, 0)$ and $e_- = \min(e_k, 0)$. Contracts held are e_+ and contracts owed are e_- . Some of the contracts held may be in default. The extent of default will be determined shortly. At present let us write $r = (r^1, r^2) \in [0, 1]^I$ as the vector of *returns ratios on preexisting contracts*. This vector will be determined, along with prices, in the temporary equilibrium. If the individual is holding preexisting contracts e_+ , and the returns ratios are r , he is actually paid

$$re_+ \equiv (r_1 e_{+1}, \dots, r_I e_{+I}).$$

Prices at date 1, to be determined in equilibrium, are denoted

$$p = (p^1, p^2) \in \Delta^I, \text{ where } \Delta^I = \{p \in R_+^I \mid \sum_k p_k = 1\}.$$

If the individual faces prices-returns (p, r) , his wealth is given by $p \cdot (\omega + re_+ + e_-)$. We shall say that the individual is *solvent* or *bankrupt at* (p, r) according to whether or not this expression is non-negative.

If the individual is bankrupt, the *default rule* modifies his endowment in the following way: he is allowed to default an equal proportion of all the contracts on which he is a debtor. This *default proportion at prices p and returns r* is given by

$$d(p, r) = \min \{ \delta \geq 0 \mid p \cdot (\omega + re_+ + (1 - \delta)e_-) \geq 0 \}.$$

This is the bankruptcy law to which we have referred in the last section. At each (p, r) the endowment of the individual is given by

$$\begin{aligned} \omega + \eta(p, r) &= \omega + re_+ + (1 - d(p, r))e_-, \\ (\eta(p, r) &= (\eta^1(p, r), \eta^2(p, r)). \end{aligned}$$

Thus the interpretation of $\eta(p, r)$ is the vector of preexisting contracts, after the bankruptcy law has been applied. By definition, the value of the altered endowment is non-negative.

We shall let the number of individuals in the economy be I and denote the set of all individuals by $S = \{1, \dots, I\}$. We denote quantities referring to a particular individual by a pre-superscript i . In order for the default rule to be feasible when applied simultaneously to all individuals, we must have that the amount of contracts actually repaid equals the

amount of outstanding obligations after the preexisting contracts have been adjusted. This can be expressed as

$$-\sum_{i \in S} (1 - d(p, r))e_{-k} = r_k \sum_{i \in S} e_{+k} \quad (10.1)$$

for all k .

If eq. (10.1) is satisfied, we will say that r is a *consistent* returns vector at prices p .

It can easily be observed that condition (10.1) may fail at an arbitrary (p, r) . The principle result of this section will be to show that for each strictly positive $p \in \Delta^I$, there is a *unique* consistent returns vector.

This result will be useful when we come to proving the existence of an equilibrium, for in searching for an equilibrium it will suffice to vary only p , letting r be determined endogenously as the unique returns vector whose existence is asserted above. A second, technical, reason for being interested in this result is that we will eventually apply a fixed point theorem to the domain of the aggregate excess demand function. For this, we will use the fact that the domain can be approximated by a sequence of compact convex sets. If, in $\Delta^I \times [0, 1]^I$, the set of consistent r at each p had a pathological shape, the application of fixed point methods would be inappropriate. The uniqueness of the consistent returns ratio vector avoids this potential difficulty.

In the model we have been discussing, debts are centralized in the sense that everyone trades with an abstract market. However, it is clear that this system is equivalent to a bilateral trading system with debts appropriated in a particular way. Suppose that, on each contract, every creditor holds the same proportion of the contracts of each debtor. Thus, the proportion of his debt on contract k that debtor i owes to creditor j is the share of total outstanding contracts on good k that i 's contracts represent.

Let $p \gg 0$ be fixed. Let the debts owed by the various individuals, when appropriated in this way, be written as a matrix $T = [t_{ij}]$, where t_{ij} is the value at prices p of the claims of individual i on individual j . Let $t_{ii} \equiv 0$.

Let W be the vector of values of naturally occurring endowments of the individuals at these prices. Let $\tau_j = \sum_{i \in S} t_{ij}$; τ_j is the total value of the claims against individual j .

Let

$$E = T + \begin{bmatrix} -\tau_1 & & 0 \\ & \ddots & \\ 0 & & -\tau_l \end{bmatrix}.$$

Thus, if in the bilateral appropriation of the debt we have chosen individuals are defaulting with proportions $d = ({}^1d, \dots, {}^ld)$, the vector of wealth post-default is given by $W + E(1 - d)$, where 1 is the vector each element of which is one. For d to be an equilibrium default proportions vector for this way of appropriating debt, we must have

$$\left. \begin{aligned} W + E(1 - d) &\geq 0 \\ 0 \leq d \leq 1 \\ (W + E(1 - d)) \cdot d &= 0 \end{aligned} \right\} \quad (10.2)$$

The interpretation of the last equation is that any individual who is defaulting on some contracts (${}^id > 0$) must have zero wealth.

It is clear that this appropriation of debt gives rise to a system that is equivalent to the original one in the following sense: if d is an equilibrium of the bilateral trade system (i.e. satisfies eq. (10.2)) and prices are p , then the returns ratios $r = (r_1, \dots, r_l)$ calculated from the equations

$$r_k = \frac{-\sum_{i \in S} (1 - {}^id) e_{-k}}{\sum_{i \in S} {}^i e_{+k}} \quad k = 1, \dots, l$$

are consistent at p . Conversely, if r is consistent at p , then the default proportions associated with (p, r) , ${}^id = {}^id(p, r)$, satisfy (10.2).

We will use this equivalence in proving the existence and uniqueness of a consistent r at each $p \gg 0$.

LEMMA 10.1. *There exists a consistent r for any $p \gg 0$.*

PROOF. Assume that, for each k , $\sum_{i \in S} {}^i e_{-k} \neq 0$. If, for some k , this sum is zero, we will be able to set $r_k = 1$ without loss of generality, since contract k will be held by no one. From the definition of ${}^id(p, r)$, it is clear that

$$1 - {}^id(p, r) = \min \left[1, \frac{p \cdot ({}^i\omega + r^i e_+)}{-p \cdot {}^i e_-} \right].$$

Thus define, for $r \in [0, 1]^l$ and $k = 1, \dots, l$,

$$F_k(r) = \frac{\sum_i \min \left[1, \frac{p \cdot ({}^i\omega + r^i e_+)}{-p \cdot {}^i e_-} \right] {}^i e_{-k}}{\sum_i {}^i e_{-k}}.$$

Because $\sum_i {}^i e_{+k} = -\sum_i {}^i e_{-k}$, it is clear that $F_k(r) \in [0, 1]$. Let $F(r) = (F_1(r), \dots, F_l(r))$. The continuity of F is obvious. Hence F has a fixed point, applying the Brouwer fixed point theorem. A fixed point of F is a consistent value of r at prices p , by definition. (Note that the condition $p \gg 0$ is used to ensure that

$$p \cdot ({}^i\omega + r^i e_+) / -p \cdot {}^i e_- \neq 0/0,$$

and hence that F is well-defined.) $Q.E.D.$

Let r^* be a consistent returns vector and let $d^* = d(p, r^*)$ be a vector of default proportions satisfying eq. (10.2). We will show that there exists no other solution to eq. (10.2). Hence r^* will be unique, except if there is some commodity on which no contracts are outstanding in which case the r_k^* corresponding to it can be set equal to 1, with no effect on any real variables.

Our proof of the uniqueness of d^* relies on two basic facts. First, aggregate wealth remains constant since the default rule only redistributes preexisting contracts. Second, any individual i with ${}^id > 0$ has zero wealth, by the third relation in eq. (10.2).

LEMMA 10.2. *The system 10.2 has a unique solution.*

PROOF. Let d^* and d' be solutions such that $d^* \neq d'$. Let $Q = \{i \mid {}^id' < {}^id^*\} \neq \emptyset$. Let $Q' = \{i \mid {}^id' \geq {}^id^*\}$. We must have that $Q' \neq \emptyset$, since otherwise ${}^id^* > 0$ for all i , every individual would be bankrupt, and aggregate wealth would be zero, a contradiction. If any member of Q is in debt to any member of Q' , the aggregate wealth of members of Q' is higher in the d' situation than in d^* . Every member of Q is bankrupt in the d^* situation, since ${}^id^* > 0$ for every $i \in Q$. Thus no member of Q is in debt to any member of Q' , for if so, the aggregate wealth of members of Q would be less than zero in the d' situation, violating the consistency of d' . Thus all preexisting contracts between members of Q and Q' involve the former as creditors and the latter as debtors. Since contracts

between two members of Q cannot affect the aggregate wealth of Q , this aggregate must be at least as great as the value of the naturally occurring endowment of Q . But this contradicts the fact that every member of Q is bankrupt in the d^* situation. $Q.E.D.$

We denote by $r^*(p)$, the unique returns ratio vector associated with $p \gg 0$. It is obvious from definitions that $r^*(\cdot)$ is continuous on $\text{int } \Delta^l$. It should be kept in mind that $r^*(\cdot)$ also depends on $\{(\omega, e)\}$ which are data of the system at date 1.

10.5. Individual Behavior

Consider an individual who is faced with prices $p \in \Delta^l$ and returns ratios $r \in [0, 1]^l$. His endowment is modified by the default rule to $\omega + \eta(p, r)$. The individual is to select an *action* which consists of a vector for current consumption, x , and a vector of purchases and sales of futures contracts, b . If $b_k > 0$, we will say that the individual has purchased a contract for delivery of commodity k at time 2, subject to the provisions of the bankruptcy-default laws; conversely, for sales, $b_k < 0$, delivery is promised subject to these rules. Note that the institutional rules of the economy enter directly into the nature of the contracts themselves.

We denote an action by $z = (x, b) \in R^l$. We assume that *consumption sets* are the non-negative orthants of the appropriate commodity space; hence $x \geq 0$. No restriction is placed on the domain of b .

We next determine the *consequences* of an action. Consequences have three components: consumption in each of the two periods, and the extent of bankruptcy, if any, in period 2.

The second period has essentially the same structure as the first. At any prices-returns combination, the individuals' endowments are modified according to the same default rules. There will, of course, be only one consistent returns vector associated with each period 2 price vector. However, since the returns vector depends on the preexisting contracts of all individuals, any one individual cannot determine what it will be if his only information is prices and his own actions. We will use $\sim$ over any letter to denote the magnitude in date 2 that corresponds to the one in date 1 represented by this letter.

An individual having taken the action $z = (x, b)$ has preexisting contracts at date 2 equal to $\eta^2(p, r) + b \equiv \tilde{e}$. If prices are $\tilde{p}$ and returns are $\tilde{r}$ and

$$\tilde{p} \cdot (\omega^2 + \tilde{r}\tilde{e}_+ + \tilde{e}_-) \geq 0,$$

the individual selects $\tilde{x} \geq 0$ such that

$$\tilde{p} \cdot \tilde{x} \leq \tilde{p} \cdot (\omega^2 + \tilde{r}\tilde{e}_+ + \tilde{e}_-).$$

If the value of his wealth is negative, so that he must default on some of his debt, he is forced to consume $\tilde{x} = 0$. The *extent of bankruptcy* y associated with this position is defined as

$$y = \tilde{d}(\tilde{p}, \tilde{r})\tilde{p} \cdot \tilde{e}_-$$

which is the value of the contracts on which he is defaulting. We define $\tilde{d}(\cdot, \cdot)$ exactly as $d(\cdot, \cdot)$.

Thus the extent of bankruptcy is non-zero only when consumption is zero, and conversely, consumption in the second period can be positive only if the individual is solvent at that date.

We assume that the individuals' attitudes regarding a consequence $(x, \tilde{x}, y)$ can be described by a von Neumann-Morgenstern utility function u . The domain of u will be all triples $(x, \tilde{x}, y)$ satisfying $x \in R_+^l$, $x \in R_+^{l_2}$, $y \in R_-$, and $y\tilde{x} = 0$, in accordance with the remarks above. One should note that the domain of u is non-convex because of the last condition. This will lead to a non-convexity in the demand correspondence and hence to the fact that we will only be able to prove the existence of an approximate equilibrium, following the methods of Starr [15], with better approximations as the number of individuals becomes large. With respect to u , we assume:

(u.1) $u(x, \tilde{x}, 0)$ is concave in its first two arguments;

(u.2) $u(x, 0, y)$ is concave in its third argument for each $x \in R_+^{l_2}$;

(u.3) u is strictly monotone in all its arguments throughout the domain; and

(u.4) u is bounded above by $\bar{u}$.

Thus a solvent individual at date 2 will choose $\tilde{x} \in R_+^{l_2}$ to maximize $u(x, \tilde{x}, 0)$ subject to

$$\tilde{p} \cdot \tilde{x} \leq \tilde{p} \cdot (\omega^2 + \tilde{r}\tilde{e}_+ + \tilde{e}_-).$$

The utility of a bankrupt individual is $u(x, 0, \tilde{d}(\tilde{p}, \tilde{r})\tilde{p}\tilde{e}_-)$.

For each action (x, b) at date 1, and every

$$(\tilde{p}, \tilde{r}) \in \text{int} \Delta^{l_2} \times [0, 1]^{l_2}$$

define $\phi(x, b, \tilde{p}, \tilde{r})$ as the value of the second period problem of an individual who has taken this action and is faced with this situation. Note that the function ϕ depends on the (p, r) faced at time 1 because $\tilde{e}$ varies with this through η .

At time 1, the individual is assumed to maximize his expected utility subject to the budget constraint that $p \cdot z \leq p^1 \cdot (\omega^1 + \eta^1(p, r))$. We describe his expectations as follows.

For each $(p, r) \in \Delta^l \times [0, 1]^l$ the expectations of the individual regarding $(\tilde{p}, \tilde{r})$ are given by the measure on $\Delta^{l_2} \times [0, 1]^{l_2}$ (with its Borel σ -field) denoted $\psi(p, r)$. Thus,

$$\psi: \Delta^l \times [0, 1]^l \rightarrow M_{\Delta^{l_2} \times [0, 1]^{l_2}}.$$

For $p^2 \neq 0$, define

$$\pi(p) = (p^2 / \sum_k p_k^2) \in \Delta^{l_2}.$$

We assume that ψ satisfies:

($\psi.1$) $\psi(\cdot, \cdot)$ is continuous in the weak topology;

($\psi.2$) for every $p \gg 0$ and $r \in [0, 1]^l$, $\pi(p) \in \text{int co supp}_\Delta \psi(p, r)$ where supp_Δ is the projection of the support on its first factor space, and int co is the interior of the convex hull of;

($\psi.3$) there exists an open set $C \subseteq \Delta^{l_2}$ such that for every $p \in \Delta^l$, $C \subseteq \text{supp}_\Delta \psi(p, r)$; the set C is assumed to be the same for every individual; and

($\psi.4$) for every $(p, r) \in \Delta^l \times [0, 1]^l$

(i) $\psi(p, r) \{(\tilde{p}, \tilde{r}) \mid \tilde{p}_k = 0 \text{ for some } k\} = 0$

(ii) $\psi(p, r) \{(\tilde{p}, \tilde{r}) \mid \tilde{r}_k = 0 \text{ for some } k\} < 1$.

We now discuss each of these conditions on ψ . Assumption ($\psi.1$) means that small changes in the environment of any individual will give rise to small changes in his beliefs. This appears to be a very reasonable condition; it has been used by all previous writers in this area. Nevertheless, we should remark in passing that it would be violated by an individual who behaves as a hypothesis-testing classical statistician. For Bayesians, however, it will follow from some continuity conditions on their underlying postulates.

Assumption ($\psi.2$) is the condition that no individual believes that any linear combination of futures contracts is *sure* to make an arbitrage profit.

Assumption ($\psi.3$) is a compatibility condition on the expectations of the various individuals. It asserts that there is some open set that is always given a positive, though perhaps very small, weight in everyone's beliefs. We will discuss this condition and the necessity for assuming it when it is used in the next section.

Assumption ($\psi.4$)(i) is the condition that no prices are ever expected to be zero. The reason for this is that, since all goods are strictly desired by the individual himself, and since the expectations concern his beliefs about future equilibria, he can be sure that no goods will be free in these equilibria. Assumption ($\psi.4$)(ii) asserts that futures contracts are never expected to be worthless because of default with certainty.

Returning to the individual's problem at date 1, he solves

$$\max_{(x, b)} \int \phi(x, b, \tilde{p}, \tilde{r}) d\psi(\tilde{p}, \tilde{r})$$

subject to

$$p^1 \cdot x + p^2 \cdot b \leq p^1 \cdot (\omega^1 + \eta^1(p, r)).$$

We define the individual's demand correspondence as follows:

$$\xi(p, r) = \{(x, b) \mid \int \phi(x, b, \tilde{p}, \tilde{r}) d\psi(\tilde{p}, \tilde{r}) \geq \int \phi(x', b', \tilde{p}, \tilde{r}) d\psi(\tilde{p}, \tilde{r})\}$$

for all (x', b') satisfying

$$p^1 \cdot x' + p^2 \cdot b' \leq p^1 \cdot (\omega^1 + \eta^1(p, r)).$$

The remainder of this section is concerned with various properties of $\xi(\cdot, \cdot)$. They are generally similar to standard results used in general equilibrium theory and we will only indicate methods or cite the work of others to conserve space.

LEMMA 10.3. For all $(p, r) \in \text{int } \Delta^l \times [0, 1]^l$, $\xi(p, r) \neq \emptyset$.

PROOF. Using the boundedness of u from above, the concavity property ($u.2$) and the no-sure-thing property ($\psi.2$), one may appeal to the result of Leland [11, theorem III], on the existence of optimal policies under uncertainty.

LEMMA 10.4. $\xi(\cdot, \cdot)$ has a closed graph.

PROOF. Using the methods of Grandmont [5], Sondermann [4] or Green [3], this can be proven directly. The proof in our case will rely on the continuity of $\eta(\cdot, \cdot)$, as well as on continuity of expectations (ψ .1) and utility, which follows from (u.1) and (u.2).

LEMMA 10.5. If $\langle p^j \rangle \in \text{int } \Delta^1$ and $p^j \rightarrow \bar{p} \in \text{bdy } \Delta^1$, then there exists an individual i such that if $z^j \in \xi(p^j, r^*(p^j))$, then $\|z^j\| \rightarrow \infty$.

PROOF. We will treat two separate cases:

- (I) $\bar{p}^2 \neq 0$, and
- (II) $\bar{p}^2 = 0$.

In case (I) standard methods apply directly, since the good that becomes free in the limit can be purchased to a greater extent and the extra expenditure can be financed with sales of a futures contract with a positive limiting value. The required financing approaches zero in the limit, and hence continuity of the expected utility index suffices to contradict the optimality of any bounded sequence of actions for any individual. Case (II) requires separate treatment, since financing increased purchases of a futures contract may be impossible through decreased purchases of current commodities as current consumption may already be zero. We therefore offer a more detailed proof in this case.

For each j , aggregate wealth is given by $p^j \cdot \sum_i \omega^i$. Since $\sum_i \omega^i \gg 0$ we have that, for j sufficiently large, there exists $\varepsilon > 0$ such that $p^j \cdot \sum_i \omega^i = I \varepsilon > 0$.

Thus there must be some individual i and some subsequence (retain the index j) such that

$$p^j \cdot (\omega^1 + \eta^1(p^j, r^*(p^j))) \geq \varepsilon$$

for all j , since otherwise aggregate wealth would converge to zero, contradicting the statement above. Consider this individual to be fixed; we shall drop the index i for brevity. Let $z^j = (x^j, b^j)$ and $z^j \in \xi(p^j, r^*(p^j))$ and assume $\|z^j\|$ is bounded. Let a subsequence be selected converging to $(\bar{x}, \bar{b})$.

Since b is bounded, $p^{2j} \cdot b^j \rightarrow 0$; hence, for j sufficiently large, $p^{1j} \cdot x^j \geq \varepsilon/2$. In particular, for some further subsequence (retain the index j), and some $\delta > 0$, there exists a current commodity k with $\bar{p}_k^1 > 0$ such that $x_k^j \geq \delta$, for all j .

Thus, for some $\alpha = (\alpha_1, \dots, \alpha_n) \in R^{k_2}$, $\alpha > 0$, there exists a sequence $x^j \rightarrow \bar{x}$ and $0 \leq x^j < x^j$ such that

$$p^j \cdot z^j = p^j \cdot (x^j, b^j + \alpha) = p^{j1} \cdot (\omega^1 + \eta^1(p^j, r^*(p^j))).$$

By continuity of the expected utility index and strict monotonicity of the utility function we obtain a contradiction; hence $\|z^j\|$ must be unbounded. Q.E.D.

10.6. Equilibrium

Let us define the aggregate excess demand, after convexification of the individual demand correspondences, as

$$\zeta(p, r) = \sum_{i \in S} \xi^i(p, r) - \left\{ \sum_{i \in S} \omega^i, 0 \right\}.$$

That is, excess demand for current commodities is demand minus supply; excess demand for futures contracts is the sum of offers to buy such contracts minus offers to sell them.

Equilibrium is defined as a (p, r) such that $0 \in \zeta(p, r)$ and r is consistent at p .

The proof of the existence of an equilibrium for this convexified excess demand function will follow essentially standard lines, although at two points the usual techniques will have to be modified. It is at these instances that we will use assumptions (u.4) (bounded utility), (u.2) (concave subjective bankruptcy penalty) and (ψ .3) (existence of a common open set that is always given positive weight).

It can be shown, given the existence of an equilibrium for the convexified demand, that an approximate equilibrium exists for the original economy. For this result, one can employ the methods of Starr [15]. An alternative approach would be to use an atomless measure space of agents, in which case an exact equilibrium exists which can be viewed as the limit of approximate equilibria for sequence of large economies converging to it in a suitably defined way; see Hildenbrand [8] and Hildenbrand, Schmeidler and Zamir [9].

10.6.1. A sketch of the proof of the existence theorem

We will now sketch the proof we shall use for the existence theorem and point out those instances at which further reasoning, of a non-standard

variety, is required. Both problems arise through the fact that, when bankruptcy is allowed, the set of allowable actions is not bounded below.

First, we observe that, by virtue of the results of section 10.4, it suffices to consider, for each $p \in \text{int } \Delta^l$, only $r^*(p)$, the unique consistent returns vector. Equilibrium could equally well have been defined as a $p \in \text{int } \Delta^l$ for which $0 \in \zeta(p, r^*(p))$. Thus we let $\langle S' \rangle$ be an increasing sequence of compact, convex subsets of $\text{int } \Delta$ such that $\bigcup_{j=0}^{\infty} S^j \supseteq \text{int } \Delta^l$. To each S^j we apply the fundamental lemma of Debreu [2] which, by virtue of lemma 10.4, implies the existence of $p^j \in S^j$ and $z^j \in \zeta(p^j, r^*(p^j))$, such that $p^j \cdot z^j = 0$ and $p \cdot z^j \leq 0$ for all $p \in S^j$.

One then must show that $\{z^j\}$ is bounded. This is the first point at which some new arguments, based on the assumptions mentioned, must be advanced. In the Arrow-Debreu model, the boundedness of z^j followed directly from $p^0 \cdot z^j \leq 0$ for some $p^0 \gg 0$, $p^0 \in S^0$, and the boundedness of the actions z from below (consumption sets bounded below). One then must prove that if $p^j \rightarrow \bar{p} \in \text{bdy } \Delta^l$, then $\{z^j\}$ is not bounded. Usually this follows directly from lemma 10.5 and boundedness of the consumption sets from below. But without this, it is possible that although excess demands for two individuals become large their sum remains bounded because their speculations on futures transactions 'cancel each other out'.

Combining these results, one has that (p^j, z^j) has a subsequence converging to (p^*, z^*) with $p^* \in \text{int } \Delta^l$. Because the excess demand correspondence has a closed graph, $z^* \in \zeta(p^*, r^*(p^*))$, and $z^* = 0$ follows from $p \cdot z^* \leq 0$ for all $p \in \text{int } \Delta^l$.

We now lay the groundwork for the two missing steps in the above argument with the following lemmas.

LEMMA 10.6. Let C be an open subset of $\text{int } \Delta^2$. There exists $C^* \subseteq C$, C^* open and a real number $\delta > 0$ such that, for any partition, $\mathcal{P}$, of the indices $\{1, \dots, \ell_2\}$ into two non-empty subsets K_+ and K_- , there exists $V_{\mathcal{P}} \subseteq C$, an open set with the property that

$$p \in V_{\mathcal{P}}, p^* \in C^*$$

implies

$$p_k - p_k^* > \delta \quad \text{for } k \in K_-$$

and

$$p_k - p_k^* < -\delta \quad \text{for } k \in K_+.$$

PROOF. Choose $p^0 \in C$ arbitrarily. Let $\beta > 0$ be such that $|p - p'| < \beta$ for all $p \in C$. Let $\varepsilon = \beta/2\ell_2$.

Thus, for any partition $\mathcal{P} = (K_+, K_-)$ where $\#K_+ = a_+$ and $\#K_- = a_-$ are the number of members of K_+ and K_- , we have

$$p'_{\mathcal{P}} = \begin{cases} p_k^0 + \varepsilon a_- & \text{for } k \in K_+ \\ p_k^0 - \varepsilon a_+ & \text{for } k \in K_- \end{cases}$$

is in C . (Note that $p'_{\mathcal{P}} \in \Delta^2$ by construction.)

Let C^* be the $\varepsilon/4$ -ball about p^0 , and $V_{\mathcal{P}}$ the $\varepsilon/4$ -ball about $p'_{\mathcal{P}}$. Let $\delta = \varepsilon/2$. $V_{\mathcal{P}}$ is clearly open and $V_{\mathcal{P}} \subseteq C$ because $|p_k - p'_{\mathcal{P}k}| < \varepsilon/4$ and $|p_k^0 - p'_{\mathcal{P}k}| < \beta/2$ imply $|p_k - p_k^0| < \beta$. To show the property stated in the lemma, choose $p \in V_{\mathcal{P}}$ and $p^* \in C^*$, drop the index $\mathcal{P}$, considering the partition to be fixed, and let $k \in K_+$ be fixed.

By definition of p' we have

$$\varepsilon \leq \varepsilon a_- = p_k^0 - p'_k.$$

Since $p \in V$ and $p^* \in C^*$ we have

$$\begin{aligned} p_k^0 &< p_k^* + \varepsilon/4, \\ p'_k &< p_k - \varepsilon/4. \end{aligned}$$

Combining the last three relations, we have

$$p_k - p_k^* < -\varepsilon/2 = -\delta.$$

Exactly analogous reasoning can be used for $k \in K_-$. Q.E.D.

LEMMA 10.7. Let T be open in C ; then

$$\psi_{\Delta}(p, r)(T) = \psi(p, r)(\{(\tilde{p}, \tilde{r}) \in \Delta^2 \times [0, 1]^{\ell_2} \mid \tilde{p} \in T\})$$

is bounded away from zero as a function of (p, r) .

PROOF. Let

$$\{(\tilde{p}, \tilde{r}) \in \Delta^2 \times [0, 1]^{\ell_2} \mid \tilde{p} \in T\} \equiv X.$$

Suppose that $\psi(p, r)(X)$ is not bounded away from zero. Then there exists a sequence $\langle (p^j, r^j) \rangle$ such that $\psi(p^j, r^j)(X) \rightarrow 0$. We can assume,

without loss of generality, that $(p^j, r^j) \rightarrow (\bar{p}, \bar{r})$. By the weak continuity of ψ (assumption $(\psi.1)$) we have

$$\lim_j \inf \psi(p^j, r^j)(X) \geq \psi(\bar{p}, \bar{r})(X),$$

since X is open in $\Delta^{l_2} \times [0, 1]^{l_2}$. (The equivalence of weak continuity to this statement may be found in Parthasarathy [12].) Therefore $\psi(\bar{p}, \bar{r})(X) = 0$. Then $\psi(\bar{p}, \bar{r})(\Delta^{l_2} \times [0, 1]^{l_2} \setminus X) = 1$. But this contradicts $(\psi.3)$ since, if it holds, we have $\psi(p, r)(\{\bar{p}, \bar{r}\}) \in \Delta^{l_2} \times [0, 1]^{l_2} \mid \bar{p} \in C(T) = 1$ which implies $\text{supp}_\Delta \psi(\bar{p}, \bar{r}) \subseteq C(T)$. This contradiction establishes the lemma. *Q.E.D.*

The index i of an individual is deleted throughout the following lemma in which the demand for futures contracts is characterized, to shorten the notation.

LEMMA 10.8. Let $z^j = (x^j, b^j)$ and $z^j \in \xi(p^j, r^*(p^j))$ for $j = 1, 2, \dots$. There exists a number B such that $\|b^j\| > B$ implies

$$\bar{p} \cdot (\omega + \eta(p^j, r^*(p^j)) + b^j) \geq 0$$

for all $\bar{p} \in C^*$, where C^* is the set whose existence is asserted in lemma 10.6.

PROOF. Suppose not. Then one can extract a subsequence diverging to $+\infty$ in norm such that, for each j , there exists $\bar{p}^j \in C^*$ and $\bar{p}^j \cdot (\omega + \eta(p^j, r^*(p^j)) + b^j) < 0$. If $\{b^j\}$ is bounded above, then $\bar{p} \cdot (\omega + \eta^j + b^j) < 0$ for all $\bar{p} \in C^*$ (where we have written η^j for $\eta(p^j, r^*(p^j))$). Since the utility function diverges to $-\infty$ as its third argument becomes large and negative (follows from $(u.2)$, $(u.3)$ and $(u.4)$), and since $\psi(\cdot, \cdot)(\{\bar{p}, \bar{r}\}) \mid \bar{p} \in C^*$ is bounded away from zero by lemma 10.7 and since u is bounded above $(u.4)$, the expected utility of these actions diverges to $-\infty$. Since $x = \bar{x} = y = 0$ is always attainable with certainty, this violates $z^j \in \xi(p^j, r^*(p^j))$ for large j . Thus $\{b^j\}$ is not bounded above.

Hence, for a subsequence which we can assume to be the original, there exists a partition $\mathcal{P} = (K_+, K_-)$ of the indices $\{1, \dots, l_2\}$ such that

$$b_k^j \rightarrow \infty \quad \text{for some } k \in K_+$$

and $\{b_k^j\}$ is bounded below for all $k \in K_+$ and

$$b_k^j \rightarrow \infty \quad \text{for some } k \in K_-$$

and $\{b_k^j\}$ is bounded above for all $k \in K_-$.

Let V_ϕ and $\delta > 0$ be as asserted in Lemma 10.6. We have that, for all $p' \in V_\phi$ and $\bar{p} \in C^*$,

$$p' \cdot (\omega + \eta^j + b^j) \leq \bar{p} \cdot (\omega + \eta^j + b^j) - \delta \|\omega + \eta^j + b^j\|$$

The first term on the right is non-positive and the second diverges to $-\infty$. Hence, using the same properties of utility and expectations appealed to in the case of $\{b^j\}$ bounded above, the expected utilities of $\{z^j\}$ must diverge to $-\infty$, contradicting $z^j \in \xi(p^j, r^*(p^j))$. This contradiction establishes the lemma. *Q.E.D.*

LEMMA 10.9. Let $\langle p^j \rangle \in \text{int } \Delta^l$ and $p^j \rightarrow \bar{p} \in \text{bdy } \Delta^l$; let $z^j \in \xi(p^j, r^*(p^j))$; then $\|z^j\| \rightarrow \infty$.

PROOF. Let

$$z^j = (x^j, b^j) = \sum_{i \in S} ({}^i x^j, {}^i b^j).$$

By lemma 10.5, there exists an individual i such that $\|({}^i x^j, {}^i b^j)\| \rightarrow \infty$. If $\|{}^i x^j\| \rightarrow \infty$ for some i , then, because ${}^i x^j \geq 0$, we have $\|x^j\| \rightarrow \infty$, and the result of the lemma holds.

Thus assume that $\{{}^i x^j\}$ is bounded for all i , hence that $\|{}^i b^j\| \rightarrow \infty$ for some i . Let $S' = \{i \in S \mid \|{}^i b^j\| \rightarrow \infty\}$. By the last lemma,

$$\bar{p} \cdot (\omega + {}^i \eta(p^j, r^*(p^j)) + {}^i b^j) \geq 0$$

for all $\bar{p} \in C^*$, and all $i \in S'$. Since C^* is open we have that

$$\bar{p} \cdot (\omega + {}^i \eta(p^j, r^*(p^j)) + {}^i b^j) \rightarrow \infty$$

for $\bar{p} \in C^*$, $i \in S'$, as j becomes large. (It is at this point that the openness of C , and hence C^* , becomes crucial). Hence,

$$\bar{p} \cdot \left(\sum_i {}^i \omega + \sum_i {}^i \eta(p^j, r^*(p^j)) + \sum_i {}^i b^j \right) \rightarrow \infty$$

for such i, j and $\bar{p}$. But $\sum_i {}^i \eta(p^j, r^*(p^j)) = 0$ because $r^*(p^j)$ is consistent at p^j by definition. Since ω is constant, the limit above implies $\sum_i {}^i b_k^j \rightarrow \infty$ for some k . *Q.E.D.*

THEOREM. There exists an equilibrium for the convexified economy.

PROOF. Let D^j be an increasing sequence of compact convex sets in $\text{int } \Delta^l$ such that $\text{int } \Delta^l \subseteq \bigcup_j D^j$. Applying the lemma of Debreu to the

convexified aggregate excess demand $\zeta(\cdot, r^*(\cdot))$ restricted to each successive D^j , we obtain the existence of a sequence $\langle p^j, z^j \rangle$ in $\Delta^l \times R^l$ such that $p^j \cdot z^j = 0$, $z^j \in \zeta(p^j, r^*(p^j))$ and $p \cdot z^j \leq 0$ for all $p \in D^j$, for each j . We first show that $\{z^j\}$, so constructed, is bounded.

If $\|x^j\| \rightarrow \infty$, then $\|b^j\| \rightarrow \infty$, for otherwise $p \cdot z^j > 0$ for some $p \in D^1$. Hence it suffices to show that $\{b^j\}$ is bounded. If not, then, by lemma 10.8,

$$\bar{p} \cdot \left(\sum_i w^i \omega + b^j \right) \geq 0 \quad \text{for all } \bar{p} \in C^*.$$

Since C^* is open,

$$\bar{p} \cdot \left(\sum_i w^i \omega + b^j \right) \rightarrow \infty.$$

Let $\bar{p}$ and $\bar{j}$ be selected such that, for some $\alpha > 0$, $\bar{p} = (\bar{p}^1, \alpha \bar{p}) \in D^{\bar{j}}$ for some $\bar{p} \in C^*$. Thus $\bar{p} \cdot z^{\bar{j}} \rightarrow \infty$ since $x^j \geq 0$. But this contradicts $\bar{p} \cdot z^j \leq 0$ for $j \geq \bar{j}$. Hence $\{z^j\}$ is bounded.

Extract a convergent subsequence $\langle p^j, z^j \rangle \rightarrow (p^*, z^*) \in \Delta^l \times R^l$ (retaining the index j). We have that $p^* \in \text{int } \Delta^l$, for if not, $\{z^j\}$ would be unbounded according to lemma 10.9. It follows that $z^* \in \zeta(p^*, r^*(p^*))$, since ζ has a closed graph. If $z_k^* < 0$ for some k and $z^* \leq 0$, then $p^* \cdot z^* \neq 0$, contradicting $p^j \cdot z^j = 0$ for all j . If $z_k^* > 0$ for some k , we obtain a contradiction to $p \cdot z^* \leq 0$ for all $p \in \text{int } \Delta$ by considering

$$p = (\varepsilon, \dots, 1 - (\ell_2 - 1)\varepsilon, \varepsilon, \dots, \varepsilon)$$

for $\varepsilon > 0$ sufficiently small, where the element $1 - (\ell_2 - 1)\varepsilon$ is in the k th place. *Q.E.D.*

COROLLARY. *There exists an approximate equilibrium for the original economy.*

The methods of Starr [15] can be used to prove this. We omit the demonstration; see also Arrow and Hahn [1], ch. 7.

10.7. Conclusion

In this paper we have explored a typical period in a model of pure exchange with the features that markets are known to reopen in every future period, and markets exist for all commodities, current and future,

at the present date. In this way, time has been incorporated explicitly into a general equilibrium theory. The model has the property that the institutional structure of sequential trading is validated by the need and choice of the agents to trade at each date. Further, the system can be viewed as generating a sequence of such temporary competitive equilibria because the results of previous equilibria will give rise to economic environments satisfying the assumptions needed for existence of an equilibrium in any given time period.

These assumptions seem to be quite mild. Further investigation along these lines will have to make stronger qualitative assumption on the basic data of the system in order to deduce results in comparative statics or limiting results on the sequence of temporary equilibria. Other open questions concern the role of firms in models of uncertainty and the incorporation of expectations generating hypotheses that will be useful in studying properties of the model. It is hoped that models of this type will provide a framework for the study of monetary theory.

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COMMENTS

*On preexisting contracts and temporary equilibria**

Bernt P. Stigum

C10.1.1. Discussion

In his paper 'Preexisting contracts and temporary general equilibrium', Green studies 'a model of general economic equilibrium over time in which the markets for trading commodities are open at every date' (cf. [3], p. 263). His main objective is to establish (1) codes of conduct, and (2) sufficient conditions for the existence of a temporary equilibrium in a model in which there is a real possibility that one or more consumers might go bankrupt.

Green's paper provides many interesting insights into the problems of modeling the behavior of individuals in a 'bankruptcy world'. But the paper also has several serious defects which should be pointed out. First, Green's model is theoretically weak for reasons detailed in sections C10.1.1.1-C10.1.1.4 below.

C10.1.1.1. Green considers a two-period exchange economy which operates in a world in which there exists one and only one state of

nature. This economy possesses in period 1 markets for current commodities and for claims on all (1) future (i.e. period 2) commodities. In period 2 it possesses markets for period 2 commodities only. Thus the institutional structure of the Green economy differs from the Arrow-Debreu economy [2, ch. 7] only in the fact that markets are allowed to open in period 2.

In the Green economy, individual consumers in period 1 entertain expectations about period 2 prices that are multi-valued with respect to the state of nature. These expectations depend both on the prices of current goods and on the futures prices of period 2 goods. Moreover they satisfy certain reasonable postulates (cf. [3], p. 276).

It is true and ought to be obvious to Green's consumers that by acting in accordance with these expectations they will end up with a suboptimal allocation of resources over time. If they instead all behaved as if the futures prices of period 2 goods equalled the present value of the true period 2 prices, then they would achieve a Pareto-optimal allocation of resources, they would not have to worry about the possibility of bankruptcy in period 2 and they could dispense with markets in period 2 altogether.

I can't help but believe that consumers in Green's economy would eventually decide to behave like ordinary Arrow-Debreu consumers rather than in the way predicted by Green.

The preceding criticism would not be a serious one if Green's objective were only to model the first-period behavior of an economy in which consumers in period 1 could not determine the price at which commodities were to be exchanged in period 2 because, say, of a lack of certain futures markets. Then Green's model could have been justified by saying that the allowance of a complete set of futures markets in period 1 enabled Green to simplify his notation without decreasing the generality of his results. However, Green is not interested just in the first-period behavior of his economy. He is in fact attempting to model the behavior of his economy over two periods. For that reason I consider it a serious criticism.

In this respect note also that if Green were to meet the criticism detailed at the beginning of the section, i.e. if he were to throw out one or more futures markets, he would simultaneously have to make drastic changes in other facets of his model. For one thing Green would have to modify his bankruptcy law since he would have no way of evaluating

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the vector $(\omega + re_+ + e_-)$ [3, p. 270] and hence of determining $d(\cdot)$ and $\eta(\cdot)$ [3, p. 270]. Actually if Green were to modify his bankruptcy law, it would improve his model. I will now give the reasons why it would improve his model.

C10.1.1.2. Green treats the concepts of bankruptcy and negative net worth as synonymous. This is all right in an Arrow-Debreu world (cf. [2, ch. 7]). It is also all right as long as we discuss consumer behavior in Arrow and Hahn's one-period model (cf. [1, ch. 3]). However, it makes no sense at all when we discuss consumer behavior in period 1 in Green's model. There, as in the real world, a consumer should not have to declare bankruptcy as long as he is able to obtain enough funds (by borrowing or otherwise) to meet his currently maturing obligations.

Just how unreasonable Green's bankruptcy law is can best be seen by considering two simple examples.

EXAMPLE C10.1. Consider a consumer in period one in Green's economy with 'naturally occurring endowment' $\omega = (\omega^1, \omega^2) > 0$, and with pre-existing contracts $e = (e^1, e^2)$. Suppose that he faces the price vector $p = (p^1, p^2)$ and the return vector $r = (r^1, r^2)$, and that

$$p^1(r^1e_+^1 + e_-^1) > 0, \quad (\text{C10.1})$$

$$p^2(r^2e_+^2 + e_-^2) < 0, \quad (\text{C10.2})$$

$$p(\omega + re_+ + e_-) < 0. \quad (\text{C10.3})$$

This consumer has no difficulty meeting his first-period obligations. Moreover, since p^2 is only a point forecast of period 2 prices, it is entirely possible that there are vectors (x^1, b) with $x^1 \geq 0$ such that

$$p^1x^1 + p^2b = p^1(\omega^1 + r^1e_+ + e_-^1), \quad (\text{C10.4})$$

and such that in the minds of all consumers in the economy the probability is greater than 0.999 that

$$\tilde{p}[\omega^2 + \tilde{r}(b + r^2e_+^2 + e_-^2)_+ + (b + r^2e_+^2 + e_-^2)_-] \geq 0, \quad (\text{C10.5})$$

where $\tilde{p}$ and $\tilde{r}$ denote the actual period 2 price and return vector respectively. Yet according to Green's bankruptcy law the consumer is bankrupt at (p, r) and his endowment must be modified to allow for this fact.

I think it is unreasonable to declare the consumer in example C10.1 bankrupt. He contracted e with the promise to meet his e^1 obligations in period 1 and his e^2 obligations in period 2. As an honorable man he should only have to care about whether there exist vectors $x^1 \geq 0$ and b which satisfy eq. (C10.4), and which satisfy eq. (C10.5) with 'sufficiently large' probability. The signs of the left-hand sides of (C10.1)–(C10.3) are irrelevant.

EXAMPLE C10.2. Consider an economy in which $\ell_1 = \ell_2 = 1$ and assume that in this economy there is a consumer with naturally occurring endowment $\omega = (\omega^1, \omega^2) = (3\frac{3}{8}, 9)$ and preexisting contract $e = (e^1, e^2) = (0, -18)$. Assume also that this consumer faces the price vector $p = (p^1, p^2) = (4/7, 3/7)$ and the return vector $r = (r^1, r^2) > 0$, and that he orders $(x, \tilde{x}, y)$ -vectors according to the values assumed by the function

$$U(x, \tilde{x}, y) = \sqrt{(x) + (1/6)\tilde{x} + (1/12)y}, \quad (x, \tilde{x}) \geq 0, y \leq 0. \quad (\text{C10.6})$$

For this consumer the fraction d which satisfies

$$(4/7)\omega^1 + (3/7)(\omega^2 + (1 - d)e_-^2) = 0 \quad (\text{C10.7})$$

is given by

$$d = 1/4. \quad (\text{C10.8})$$

The fraction $\tilde{d}$ which satisfies

$$\min_{0 \leq \tilde{d} \leq 1} \tilde{p}[\omega^2 + \tilde{r}(b + (1 - d)e_-^2)_+ + (1 - \tilde{d})(b + (1 - d)e_-^2)_-] \geq 0 \quad (\text{C10.9})$$

is given by

$$\tilde{d} = \begin{cases} 0 & \text{if } b \geq 9/2 \\ [(9 - 2b)/(27 - 2b)] & \text{for } b < 9/2. \end{cases} \quad (\text{C10.10})$$

Moreover, if $E\{\cdot | p, r\}$ denotes the conditional expectation of $(\cdot)$ given (p, r) , and if when p and r are as above,

$$E\{(\tilde{p}, \tilde{r}) | p, r\} = (1, a), \quad (\text{C10.11})$$

where $0 < a < 1$, then it is easy to show that the consumer's indirect first-period utility function and its partial derivatives are given by

$$F(x, b, p, r) \equiv E(\phi(x, b, \tilde{p}, \tilde{r}) | p, r) \\ = \begin{cases} \sqrt{x} - [(9 - 2b)/24] & \text{for } b \leq 9/2, x \geq 0 \\ \sqrt{x} + (1/6)(b - 9/2) & \text{for } 9/2 < b \leq 27/2, x \geq 0 \\ \sqrt{x} + (1/6)(9 + a(b - 27/2)) & \text{for } b > 27/2, x \geq 0 \end{cases} \quad (\text{C10.12})$$

$$(\partial F / \partial x)(x, b, p, r) = 1/2\sqrt{x}, x \geq 0, \quad (\text{C10.13})$$

$$(\partial F / \partial b)(x, b, p, r) = \begin{cases} 1/12 & \text{for } b \leq 9/2, x \geq 0 \\ 1/6 & \text{for } 9/2 < b \leq 27/2 \\ a/6 & \text{for } b > 27/2. \end{cases} \quad (\text{C10.14})$$

With a little algebra it follows easily from eqs. (C10.13) and (C10.14) that

$$(x, b) \equiv (81/4, -45/2) \quad (\text{C10.15})$$

maximizes $F(x, b, p, r)$ subject to the constraint

$$(4/7)x + (3/7)b = 27/14. \quad (\text{C10.16})$$

The important point to notice about example C10.2 is that at *the beginning of period 1* the consumer has a second-period debt of 18 units of e^2 which (since his net worth is negative) is considered extravagant and is promptly written down so that it equals $27/2$ units of e^2 . At *the end of period 1* the consumer owes $45/2 + 27/2 = 36$ units of e^2 ! Moreover, he knows that he will be bankrupt in period 2 with probability 1. I think that a code of conduct and a model which allow such a situation to arise are unreasonable. Note that the model in example C10.2 does not violate any of Green's assumptions except the assumption that $U \leq \bar{U}$ [3, p. 275, assumption (u. 4)] which is unimportant for the example.

The reader might think that I picked an unreasonable pair of prices (p^1, p^2) in example C10.2, one which could never represent equilibrium prices in Green's economy. That this is not so can be seen from example C10.3 below. But first one more serious criticism of Green's model.

C10.1.1.3. A temporary equilibrium in Green's economy is a vector

$$(p, r, (x^1, b)^1, \dots, (x^I, b)^I),$$

where p is a strictly positive price vector and r is a return vector, and where for each $i(x^1, b)^i, i = 1, \dots, I$, is a vector of period 1 commodities

and claims on period 2 commodities which consumer i demands at (p, r) and which satisfies the equations

$$\sum_{i=1}^I (x^1, b)^i = \sum_{i=1}^I (\omega^1, 0)^i. \quad (\text{C10.17})$$

In such an equilibrium consumers can be partitioned into three groups N_1, N_2, N_3 with the following properties. Consumer i is in N_1 if and only if he believes that he will be solvent with probability 1 in period 2. Consumer j is in N_2 if and only if he believes that he will go bankrupt with probability greater than 0 but less than 1. Consumer k is in N_3 if he believes that he will go bankrupt in period 2 with probability 1. One or two of these three groups may be empty. What is important to note is that Green's assumptions do not preclude the possibility that N_3 might contain one or more consumers.

It seems to me unlikely that institutional arrangements in practice would be such that a consumer would be allowed to buy a vector (x^1, b) if it entailed his going bankrupt with probability 1 in period 2. Thus Green's model which permits temporary equilibria that allocate such 'bankruptcy pairs $(x^1, b)^i$ ' to one or more consumers appears unreasonable to me.

Here is an example which illustrates what I have in mind.

EXAMPLE C10.3. Consider an economy in which there are two consumers A and B and in which $\ell_1 = \ell_2 = 1$. Consumer A has 'naturally occurring endowment' $\omega_A = (5, 5)$, initial holdings of securities $e_A = (-10, 0)$, and utility function

$$U_A(x, \tilde{x}, y) = \sqrt{x} + (1/16\sqrt{5})\tilde{x} + (1/16\sqrt{5})y, (x, \tilde{x}) \geq 0, y \leq 0. \quad (\text{C10.18})$$

Suppose consumer A believes that, regardless of the value of (p, r) , $(\tilde{p}, \tilde{r}) = (1, 1)$ with probability 1. His indirect utility function is then given by

$$F_A(x, b, p, r) = \sqrt{x} + (1/16\sqrt{5})(5 + b), x \geq 0, -\infty < b < \infty. \quad (\text{C10.19})$$

B has 'naturally occurring endowment' $\omega_B = (15, 5)$, initial holdings of securities $e_B = (10, 0)$, and utility function

$$U_B(x, \tilde{x}, y) = x + 10\sqrt{(\tilde{x}) - y^2}, (x, \tilde{x}) \geq 0, y \leq 0. \quad (\text{C10.20})$$

Suppose, consumer B too believes that, regardless of the value of (p, r) , $(\tilde{p}, \tilde{r}) = (1, 1)$ with probability 1. Then his indirect utility function is given by

$$F_B(x, b, p, r) = \begin{cases} x + 10\sqrt{(5+b)} & \text{for } b \geq -5, x \geq 0, \\ x - (5+b)^2 & \text{for } b < -5, x \geq 0. \end{cases} \quad (\text{C10.21})$$

It is easy to show that there is one and only one temporary equilibrium in the economy, namely

$$(p, r, x_A, b_A, x_B, b_B) = (4/5, 1/5, 5/8, 20, -85, 0, 85), \quad (\text{C10.22})$$

with $d = \frac{3}{8}$. In this temporary equilibrium consumer A is allocated $(x_A, b_A) = (20, -85)$. Since $\omega_A^2 = 5$ he will start out period 2 with net worth equal to -80 and will be bankrupt with probability 1.

Example C10.3 displays an economy which has a unique temporary equilibrium during period 1. This equilibrium allocates a vector (x^1, b) to one consumer which entails that he will be bankrupt with probability 1 in period 2. The model describing this economy, therefore seems to me to be unreasonable. Yet it satisfies all of Green's assumptions except $U_i(\cdot) \leq \bar{U}$, $i = A, B$.

Note also that, in the temporary equilibrium portrayed in eq. (C10.22), consumer A borrows (i.e. promises to supply) $8\frac{1}{2}$ times the available supply of the second-period commodity. Yet he believes that $\tilde{r} = 1$ with probability 1. This too seems unreasonable. If a consumer takes a position in (x^1, b) which entails that he will go bankrupt with probability 1, he should also know that $\tilde{r} < 1$ with probability 1.

If one wants to avoid temporary equilibria of the sort portrayed in eq. (C10.22) one must impose lower bounds on b . In example C10.3 a natural lower bound would be $b \geq -5$ for both A and B. In a more general economy with $\ell_2 > 1$ such bounds are not as easily derived. One possibility would be to insist that, for each and every consumer

$$p^2(b + \eta^2(p, r) + \omega^2) \geq 0.$$

Such a bound would certainly make sense in Green's model since then Green's assumption ($\psi.2$) (cf. [3, p. 276]) would ensure that each consumer would choose (x^1, b) in such a way that he would believe that he would be solvent in period 2 with positive probability.

In my own work on temporary equilibria (which preceded Grandmont's and Arrow and Hahn's work by several years). I have often insisted that consumers in choosing (x^1, b) observe an inequality of the form $b \geq a$, where a is an exogenously determined vector which may vary from one consumer to another. By choosing a judiciously, one should be able to avoid a situation in which people in one period can borrow so much that they will go bankrupt the next period with probability 1.

So much for temporary equilibria and bankruptcy. Next a comment on Green's first-period budget constraint.

C10.1.1.4. Green insists that a consumer is bankrupt if

$$p(\omega + re_+ + e_-) < 0. \quad (\text{C10.23})$$

Here the left-hand side involves both the consumer's first and second-period 'naturally occurring endowments' $\omega = (\omega^1, \omega^2)$. It also involves both his first and second-period holdings of securities $e = (e^1, e^2)$. Yet the consumer's first-period budget constraint is given by

$$p^1 x^1 + p^2 b \leq p^1(\omega^1 + \eta^1(p, r)), \quad (\text{C10.24})$$

where the right-hand side gives the value of his first-period endowment after the bankruptcy law has been applied.

I think (C10.23) and (C10.24) are incongruous, and I am quite sure that no one consumer in Green's economy would accept (C10.24) as his first-period budget constraint. Certainly, since securities in Green's model are traded in an abstract market, consumers cannot distinguish between e^2 and b . Therefore $p^2 \eta^2(p, r)$ should appear on the right-hand side of (C10.24). Moreover, as long as Green insists that (C10.23) provides the ultimate test of whether a consumer is bankrupt in period 1, $p^2 \omega^2$ presumably ought to appear on the right-hand side of (C10.24). Consequently, the 'right' budget constraint faced by each consumer in period 1 is not (C10.24) but instead

$$p^1 x^1 + p^2 b \leq p(\omega + \eta(p, r)). \quad (\text{C10.25})$$

If we were to modify Green's model by substituting (C10.25) for (C10.24), we would also have to change the consumer's second-period budget constraint to

$$\tilde{p}\tilde{x} \leq \tilde{p}(\tilde{r}b_+ + b_-).$$

Moreover, we would have to change the aggregate excess demand function to

$$\xi(p, r) = \sum_{i=1}^I (\bar{x}_i^e(p, r) - \bar{\omega}_i).$$

Other than that the statements and proofs of Green's results would with only obvious modifications still hold.

The preceding criticisms explain my major reasons for thinking Green's model is unsound. In addition I have three minor comments concerning Green's paper:

(1) Consumers in Green's model are penalized for being bankrupt in period two, but not for being bankrupt in period 1. At least, so it seems to me since it is perfectly possible to be bankrupt in period 1 and be solvent in period 2. I do not understand why it is worse to be bankrupt in period 2 than in period 1.

(2) Green's discussion of previous work on temporary equilibria is misleading. The idea of a temporary equilibrium was first introduced in a 'formal general equilibrium model' (cf. [3], p. 263) in my paper on 'Competitive equilibria under uncertainty' [4], which was privately circulated in 1966, presented at the Winter Meetings of the Econometric Society in Chicago in 1968 and published in the *Quarterly Journal of Economics* in November 1969. Grandmont's model is a special case of my model which considers only one state of nature, one security, two periods, and no initial endowments of preexisting contracts. Moreover, sufficient conditions for the existence of a temporary equilibrium in a bankruptcy world were first given in my paper 'Resource allocation under uncertainty' [5] which was presented at the Winter Meetings of the Econometric Society in New York in 1969 and published in the *International Economic Review* in October 1972.

(3) Green's statement about the fulfilment of contracts in an Arrow-Debreu world (cf. [3], p. 266) seems incorrect to me. Since Debreu does not make any assumption about how consumers-order commodities in future periods, Green's assertion that 'even if it were possible to reopen' markets in future periods, 'no-one would want to engage in trade at the equilibrium prices that would emerge' cannot be verified.

To conclude my discussion of Green's paper, I will give a simple example of an economy which satisfies Green's assumptions and yet

does not possess a temporary equilibrium. The example shows why Green's seemingly weak result about the existence of ε -equilibria cannot be improved upon.

EXAMPLE C10.4. Consider an economy in which there are two consumers A and B and in which $\ell_1 = \ell_2 = 1$. Assume that ω_A , e_A , and $U_A(\cdot)$ are as in example C10.3. Moreover, assume that

$$U_B(x, \tilde{x}, y) = x + (1/4)\tilde{x} - y^2, (x, \tilde{x}) \geq 0, y \leq 0, \quad (\text{C10.26})$$

and that ω_B and e_B are as specified in example C10.3. Finally, assume that both A and B expect that $(\tilde{p}, \tilde{r}) = 1$ with probability 1 regardless of the observed value of (p, r) . Then A's indirect utility function is given by eq. (C10.19), and B's is given by

$$F_B(x, b, p, r) = \begin{cases} x + (1/4)(5 + b) & \text{for } b \geq -5, x \geq 0, \\ x - (5 + b)^2 & \text{for } b < -5, x \geq 0. \end{cases} \quad (\text{C10.27})$$

Using Green's bankruptcy law it is easy to see that with $r = \min(\frac{1}{2}(1 + (p^2/p^1)), 1)$ A's optimal choice of (x, b) is given by

$$(x_A, b_A)(p^1, p^2) = \begin{cases} (320(p^2/p^1)^2, -320(p^2/p^1) - 5) & \text{for } (p^2/p^1) \leq 1, \\ \text{and} \\ (320(p^2/p^1)^2, -320(p^2/p^1) - 5(p^1/p^2)) & \text{for } (p^2/p^1) > 1. \end{cases} \quad (\text{C10.28})$$

Moreover B's optimal choice of (x, b) is given by

$$(x_B, b_B)(p^1, p^2) = \begin{cases} (0, 20(p^1/p^2) + 5) & \text{if } (p^2/p^1) \leq \alpha \equiv 0.249827 \\ ((p^2/p^1)^2/2 + 10(p^2/p^1) + 20, -5 - (p^2/p^1)/2) & \\ \text{if } \alpha \leq (p^2/p^1) \leq 1, \text{ and} \\ ((p^2/p^1)^2/2 + 5(p^2/p^1) + 25, -5 - (p^2/p^1)/2) & \\ \text{if } (p^2/p^1) > 1. \end{cases} \quad (\text{C10.29})$$

From eqs. (C10.28) and (C10.29) it follows that this economy does not have a temporary equilibrium in period 1.

One more look at (C10.28)-(C10.29) will show that A's choice of (x, b) is uniquely determined for all pairs (p^1, p^2) . B's choice of (x, b) is uniquely determined for all pairs (p^1, p^2) such that $(p^2/p^1) \neq \alpha$. When $(p^2/p^1) = \alpha$, B is indifferent between $(x, b) = (0, 85.05)$ and $(x, b) = (22.53, -5.12)$. Thus if we were to convexify A's and B's demand correspondences as Green does, we would find that the 'convexified'

economy possessed one and only one temporary equilibrium in period 1, namely

$$(p, r, x_A, b_A, x_B, b_B) = ((1 + \alpha)^{-1}, \alpha/(1 + \alpha), (1 + \alpha)/2, 19.97, -84.94, 0.03, 84.94). \quad (C10.30)$$

References

- [1] Arrow, K. J. and Hahn, F. H. *General Competitive Analysis*. Holden Day: San Francisco (1971).
- [2] Debreu, G. *Theory of Value*. Wiley: New York (1959).
- [3] Green, J. Preexisting contracts and temporary general equilibrium. In this volume, pp. 263-286.
- [4] Stigum, B. P. Competitive equilibria under uncertainty. *Quarterly Journal of Economics*, **83** (1969), 533-561.
- [5] Stigum, B. P. Resource allocation under uncertainty. *International Economic Review* (October 1972).

REPLY TO COMMENTS

Jerry Green

The remarks by Professor Stigum can be classified into three broad areas: methodological criticisms concerning the structure of a model in which there are future markets in the present that reopen at a later date, objections to the definition of bankruptcy, and criticisms of the budget constraint. I will take issue with each of these and will try to indicate the reasons for the path I have followed in the paper.

At the top of p. 287, the discussant states that '... thus the institutional structure of the Green economy differs from the Arrow-Debreu economy *only* in the fact that markets are allowed to be open in period 2' [my italics]. This is correct, but the discussant has not perceived that many of the generalizations of the Arrow-Debreu model which are currently being studied lead to the institution of sequential trading. My paper, while not addressing any of these generalizations directly, has tried to recast the institutional structure of general equilibrium theory in such a way that it will be able to handle, in a consistent fashion, some of the

more complex phenomena which cannot be modeled in the Arrow-Debreu framework. Among the motivations for adopting the alternative structure, we can cite the work of Hahn on transactions costs, Radner on differential information, and the numerous contributions to the overlapping generations' literature. If any of these are present in the Arrow-Debreu system, there may be a mutual incentive for agents to reopen markets at later dates, as the economy evolves in time. This reopening will cause them to recast their decision problem at the initial date and it is on this re-evaluation of the equilibrium at the initial date that I have tried to focus.

Professor Stigum's comments on *ex post* efficiency and his insistence that individuals would realize that they would be better off with the Arrow-Debreu framework indicate that he believes my model to be a substitute for the Arrow-Debreu model in the same economic environment. I do not view it in this way; and it is in fact only the differences in the economic environment that can justify the difference in the institutional structure I have chosen. However, rather than introduce many things at once, I tried to pose a model as close as possible to the Arrow-Debreu pure-exchange model, but with sequential trading possible. This would focus on the differences in the institutional framework as distinct from those introduced by additional complexities in the systems.

The discussant thinks that using a different definition of bankruptcy would improve the model. The proposal is to say that an individual is bankrupt if he cannot meet his currently maturing obligations 'by borrowing or otherwise'. But this will surely lead to inconsistencies, for, *since the market is anonymous*, everyone would carry vast amounts of debt throughout their lifetimes, consuming at unrealistically high levels. The proposed definition might be a good one in the context of a bilateral trading model where credit can be rationed according to the individual's asset position, as I suggest in section 10.3 of my paper. This criticism of the bankruptcy law should really be a criticism of the anonymity of the market, and one with which I would certainly agree. We are in need of a good bilateral general equilibrium model, but let us not put the burden of the inadequacy of a market equilibrium model where it does not belong.

One should also note that adopting Professor Stigum's suggestion in this regard would force us to declare individuals bankrupt if they could

not meet current obligations regardless of the fact that they might be able to borrow against their (large) future naturally occurring endowments to regain solvency – and they would be able to honor these commitments with certainty.

In his example C10.1, the discussant thinks that p^2 is a 'forecast' of $\tilde{p}$, but this is clearly incompatible with '... all consumers in the economy [believing] the probability is greater than 0.999 that

$$\tilde{p}[\omega^2 + \tilde{r}(b + r^2 e_+^2 + e_-^2)_+ + (b + r^2 e_+^2 + e_-^2)_-] \geq 0.$$

If this were the case, speculative forces would have lead to p^2 being a disequilibrium. I believe, therefore, that the argument of the example is inconsistent.

I see nothing pathological with example C10.2. The consumer believes that he will certainly be bankrupt in period 2 and takes this into account through the subjective (non-economic) bankruptcy penalty. The concavity of this penalty function insures us that his actions will be determined at any p , regardless of his expectations (if they fulfil the assumptions stated in my paper). They are *not a priori* bounded from below, as the discussant suggests I assume – and on which he states that he has 'often insisted' (p. 292). It is just such an assumption, grounded on neither rationality nor institutional restrictions, that I tried so hard to avoid. Actions in my model are *not* bounded below *on purpose*. Overcoming the technical difficulties created by this desire for a lack of *a priori* non-economic bounds, unlike the corresponding conditions in the Arrow–Debreu model which are well motivated, was one of the major goals of the paper.

In discussing his example C10.3, the discussant has lost sight of the competitive character of the model. If an individual knows that he will certainly be bankrupt, then, it is true that he can infer that $\tilde{r}_k < 1$ for all commodities in which he is a net debtor. But this gives him no reason to believe that $\tilde{r}_k$ will be any different for the *other* commodities and it is only these that are relevant to him. Thus, the discussant's comments in the second paragraph following this example seem to me completely irrelevant and misleading.

Professor Stigum says that I use a bankruptcy constraint that is inconsistent with the first-period budget constraint. This view is mistaken. There are two markets in period 1 – the market for period 1 commodities and that for contracts for claims on period 2 goods (with

a bankruptcy clause in the contract). The consumer's excess demand in these two markets at date 1 is simply, $\omega^1 + \eta^1(p, r) - x^1$ and b , respectively. Hence the budget constraint, $p^1 x^1 + p^2 b \leq p^1(\omega^1 + \eta^1(p, r))$. The quantities ω^2 and $\eta^2(p, r)$ have nothing to do with *market excess demands at date 1* and therefore do not enter into the budget equation. They do play a role in determining the individual's wealth position and hence bankruptcy status, which we have argued above should be related in the indicated manner in an anonymous market model. Hence they appear in the bankruptcy relation.

If we were to follow Professor Stigum's suggestions at the foot of p. 293, we would be led to an inconsistency. Consider a consumer who desires simply to consume his endowment at all prices and for whom $\eta(p, r) = 0$. If his budget constraints were (i) $p^1 x^1 + p^2 b \leq p \cdot (\omega + \eta(p, r))$ in period 1, and (ii) $\tilde{p}x \leq p(\tilde{r}b_+ + b_-)$ in period 2, he would have to choose $x^1 = \omega^1$ and $b = \omega^2$, according to (i). But then his contracts held in period 2, b_+ , would be subject to the market rate default even though he traded with no-one!

My model tried to make a distinction between preexisting contracts with others, which are subject to default, and naturally occurring endowment which is certain. Adopting Professor Stigum's modifications would necessitate destroying this feature of the model, which I believe to be a desirable one.

I do not include a 'bankruptcy penalty' in period 1 because, from the individual's point of view, this is a datum and not a choice variable at the equilibrium prices. I never meant to imply that bankruptcy in period 1 is better than in period 2 – but only that, if it does not affect the period 1 excess demand function, we need not consider it explicitly. I did not want to crowd an already messy set of notations and hence did not make explicit reference to this utility loss.

Although I realize that alternative concepts of bankruptcy are possible, I have not been successful in finding any others which are internally consistent and preserve the economic phenomena that I tried to model.