

ON THE THEORY OF EFFECTIVE DEMAND*

All of the recent research on macroeconomic equilibrium under temporarily rigid prices has recognised the central role of effective demands. Loosely speaking, effective demands are the quantities individuals desire to trade in one market when they take account of the fact that some of their trades on other markets may be constrained. Effective demands that are not satisfied could be used along with other variables as an explanation of price changes. Because this type of macroeconomic theory has taken prices as given within the period, it is necessary to be precise about the treatment of excess effective demands in order to close the model.

Although several different definitions of effective demand have been given, they do not provide a satisfactory foundation for such a theory of price change.

On one hand, we have the formalisation of quantity-constrained equilibrium of Drèze (1975). Each agent is assumed to maximise his utility facing fixed upper bounds on both purchases and sales. An equilibrium is a system of such constraints which, if perceived, would be verified in that total purchases would equal total sales for every commodity. In equilibrium it is required that only one side of any market be constrained. The aggregate supply equals the aggregate demand. Even though in the absence of constraints one side would change its announced quantities, we do not know the extent of the change and cannot compute it from the description of the equilibrium. The pressure for price changes resulting from disequilibrium cannot be measured with this concept of effective demand and rationing.

On the other hand, there are a variety of related definitions of effective demand formulated by Benassy (1973), Barro and Grossman (1975) and Malinvaud (1977), following the lead of Clower (1965). The agent faces upper bounds on trades in all markets. Effective demand is defined by a separate procedure for each commodity. To find the effective demand for a commodity the constraint limiting purchases on that market is deleted, individuals re-maximise subject to the remaining constraints, and the new value of demand for this commodity is called its effective demand. These have been called the 'Clower demands'.

The procedure used to define excess demands is quite artificial. It does not correspond to any action that would actually be taken by economic agents in the pursuit of their own ends. Therefore, although it satisfies our desideratum of producing a disparity between quantities offered to the market and those actually realised as equilibrium exchanges, it does so only by defining these

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offers on a somewhat arbitrary and hypothetical basis. They do not correspond to observable phenomena that could actually motivate price changes.¹

At any fixed price system, both definitions of effective demand will produce identical equilibria.² But neither provides a satisfactory structure in which excess effective demands emerge from the short-run quantity equilibration process at fixed prices.

In this paper we will study a concept of effective demand that does have this property. To introduce the basic ideas it is useful to define the concept of a rationing scheme. A rationing scheme for one agent is a mapping from the trades offered to the market by all agents to constraints on his trade. It is said to be manipulable³ if the value of the constraint is sensitive to the agent's own trade offer. Manipulable, deterministic rationing schemes, that is those for which realised trades are sensitive to effective demands even when the constraints are binding, are inconsistent with the attainability of a quantity constrained equilibrium, as Benassy has shown. Briefly, this is due to the fact that in riskless environments where agents know the rationing schemes they are facing, they can simply compute the level of trade offered which would cause them to be constrained to the true desired level. Manipulability of the rationing scheme insures that they can distort their professed demands to achieve any outcome. Attention is therefore directed at rationing mechanisms in which the realised trades are stochastic. Distortions will not necessarily lead to the optimal trade, and cannot be employed without risking potentially undesirable outcomes. Svensson (1977), noting the problems with effective demand concepts under deterministic rationing discussed above, has studied the stochastic analogue of non-manipulable rationing. He explores the demands of an agent who faces the probability of being rationed to a fixed, predetermined maximum trade in each market. Svensson's pioneering work has led to important insights in the theory of effective demand. However, we show in this paper that the type of non-manipulable stochastic rationing he considers is incompatible with attaining a feasible equilibrium. The realised quantities purchased and sold on each market will not be equated, even in a statistical sense.

We explore a particular manipulable stochastic rationing scheme which produces a theory of effective demand having the three desiderata discussed in this introduction:

- (1) Effective demands arise from the solution to agent's actual maximisation problems.
- (2) Effective excess demand is typically non-zero in a short-run equilibrium.
- (3) In a large economy, the actual trades realised by the rationing mechanism in a short-run equilibrium are feasible in the sense that their expected values are zero in every market and the discrepancy from true feasibility is negligible on a per capita basis.

We show that by imposing some reasonable assumptions on the behaviour of the rationing scheme, we can get a very strong characterisation of its dependence.

¹ This point has been noted previously by several writers including Svensson (1977) and Gale (1977).

² See Benassy (1977).

³ See Grandmont (1977) for a detailed discussion and precise definitions of manipulable and non-manipulable rationing schemes.

on individuals' announced trades. These implied properties may be¹ useful in deriving properties of the equilibria of a model in which all agents are faced with stochastic rationing of this type.

The paper proceeds as follows:

Section I describes the stochastic rationing scheme. It is shown that no other form of rationing scheme can satisfy the three properties above. Thus, the manipulability and stochastic nature of the rationing scheme we study are necessary consequences of the type of macroeconomic theory we are trying to build, rather than being ad hoc assumptions.

Section II presents a variety of examples of particular processes for matching offers to buy and sell and explores the resulting stochastic realisation functions. In this way we develop some of the implications of the assumptions used in the previous section to characterise consistent realisation schemes.

In Section III the theory of individual behaviour is explored. Conditions for existence and boundedness of effective demand functions are given. The problem of the existence of an equilibrium is discussed briefly in Section IV.

I. CHARACTERISING COMPATIBLE REALISATION SCHEMES

The individual economic agent at a given moment in time faces the problem of selecting his effective demands. Effective demand for a commodity is the amount he offers to trade at the prevailing market prices. These offers are not necessarily compatible with those of other agents, and therefore may not be fulfilled in the short-run situation we are considering. The agent's beliefs about the extent of the trades that he will be able to complete constitute an important datum of his decision process.

There are N markets on which offers to buy and sell commodities are made. They are assumed to be independent in the sense that the state of one market does not directly affect the realisation of trades on any other. Therefore we will temporarily concentrate on a particular market, that for commodity n . It is useful to interject, however, that interactions between markets are the essence of the phenomena we are hoping to describe; they will take a more indirect form.

Market n is not to be conceptualised as a central clearing-house, as in traditional economic analyses. Trade is highly decentralised. Each economic agent, i , offers an effective demand, z_{in} , for trade. But rather than being pooled and matched to the maximum possible extent, the realisation of trades reflects an incomplete or imperfect matching process. There may be some unsatisfied demand coexisting with unsatisfied supply. Search activity may reduce the extent of this mutual imbalance, but time is not sufficient for it to be eliminated.

The second characteristic of the realisation process is its stochastic nature, at least from the economic agent's point of view. The precise network of trading proposals may be unknown to the agent even if his information about the state of aggregate demands and supplies is very good.

Realisations are at most equal to the offers, and are always of the same sign.

¹ See Honkapohja and Ito (1979).

In this respect we have followed the traditional assumptions in the modern disequilibrium literature.

In its most general form, the actual trade of an economic agent is a random function of the offers of all other agents. We will take a simpler hypothesis: *The realisation is a random function of his own effective demand and the aggregate effective demand and supply on the market.*

Assume that there are I agents whose effective demands are z_{in} , $i = 1, \dots, I$. Aggregate effective demand is

$$Z_n^+ = \sum_{i=1}^I \max(z_{in}, 0)$$

and aggregate effective supply (defined negatively) is

$$Z_n^- = \sum_{i=1}^I \min(z_{in}, 0).$$

This section is concerned with the characterisation of the realisation functions satisfying certain conditions. As we are treating the behaviour of a single market, n , in isolation, the subscript n can be dropped for simplicity of notation. As described above, the rationing scheme is a random function of the form

A (o) $x_i = \phi_i(z_i, Z^+, Z^-)$, satisfying

A (i) $|\phi_i| \leq |z_i|$ with probability 1,

A (ii) $z_i \phi_i \geq 0$ with probability 1,

A (iii) $\sum_{i=1}^I E \phi_i(z_i, Z^+, Z^-) = 0$ for all effective demand vectors $\mathbf{z}$.

A (iv) The distribution of ϕ_i is the same for all $i = 1, \dots, I$, for each value of the arguments.

A (v) The distribution of $\phi_i(z_i, Z^+, Z^-)$ is weakly continuous in its arguments.

The requirements (i) and (ii) embody the idea that no agent can be forced to trade more than he would like, nor can he be forced to buy when he wants to sell, or vice versa.

Condition (iii) represents the feasibility condition for the rationing mechanism. The random functions ϕ_i , $i \in I$ are required to balance sales and purchases in the mean. This is obviously a weaker postulate than the real closedness of the system would imply, namely that $\sum \phi_i \equiv 0$. There are two reasons for not taking this approach. First, it would mean that the I th realisation function is functionally related to the others. This restriction might come into conflict with the fourth condition, risking vacuity of the theory, or implying restrictions on the nature of the distribution of the ϕ_i that have not been stated. Second, and more important, we have in mind the application of these postulates in a large decentralised system. Independence of the realisations together with condition (iii) will imply that the aggregate excess effective demands will be negligible on a per capita basis.

Condition (iv) is an anonymity requirement. It is related to the idea of a uniform rationing scheme (see Drèze, 1975). But we have not assumed that the

constraints are always identical. They have the same distribution only if the agents' offers are equal.

Condition (v) is a technical requirement of the usual sort.

One important condition used throughout the literature on quantity constrained equilibria is missing. This is the restriction that only one side of the market is rationed – namely, the 'long' side, that having the higher absolute value of aggregate effective demand. The justification for this omission is given by the 'imperfect matching' argument at the beginning of this section. Its history goes back to Beveridge (1930), whose primary interest was in the imperfect matching processes in the labour market, producing a coexistence of vacancies and involuntary unemployment: a 'Beveridge curve'.

Two points should be made here before we pass on. First, the results of this section are fully compatible with a complete fulfilment of effective excess demands by the short side of the market. The reason that we have not required it is that it is not necessary for our present conclusions. A non-degenerate 'Beveridge curve' may be valuable in aggregative models for use in macro-economic analysis, see e.g. Muellbauer (1977). Second, the models of equilibrium in quantity rations determine a self-consistent pattern of trades and are true equilibrium theories in this respect. They do not describe the behaviour of the system when trades are not consistent with perceived quantity constraints. Presumably, just as in any *tâtonnement* theory, trial values of quantities adjust until feasibility is reached, and only then are trades actually performed. The short-sided trading hypothesis embodies this concept well: all mutually desirable exchanges should be consummated if sufficient time were available. The present theory is of an entirely different sort. The quantities perceived as market aggregates by the agents need not be correct in order for them to compute their trial optimal actions. The realisation rule could achieve a feasible outcome given any vector of effective demands. Although we concentrate later on the case in which aggregates are correctly perceived, this is not necessary to define the allocation determined by this theory, as the short-sided rule is not imposed.¹

We now proceed to the main theorem of this section, a characterisation of the allowable rationing schemes.

Theorem

Let $I \geq 3$. If the functions ϕ_i depend upon $z_i, Z^+,$ and Z^- as stated in A (o), then under conditions A(i), A(iii), A(iv) and A(v), they can be written as

$$\begin{aligned} \phi_i(z_i, Z^+, Z^-) &= z_i \bar{s}_i^+(z_i, Z^+, Z^-) \quad \text{for } z_i \geq 0 \\ &= z_i \bar{s}_i^-(z_i, Z^+, Z^-) \quad \text{for } z_i < 0 \end{aligned}$$

where $\bar{s}_i^+$ and $\bar{s}_i^-$ are random functions whose mean is independent of z_i .

Remark. This theorem is obviously equivalent to the statement that the function $\bar{\phi}_i(z_i, Z^+, Z^-) = E \phi_i(z_i, Z^+, Z^-)$ be linear in its first argument, over the positive and negative half-lines, but perhaps with different slopes. To show that $\bar{\phi}_i$ is linear we use the following result:²

¹ See the discussion on p. 347 below which expands upon this point.

² See Hardy, Littlewood and Polya (1934), pp. 70-4.

A function f is called *mid-point convex* (concave) if for any x, y ,

$$\frac{1}{2}f(x) + \frac{1}{2}f(y) > (<) f\left(\frac{1}{2}x + \frac{1}{2}y\right).$$

If f is midpoint convex and continuous, then f is convex.

Proof of Theorem. Consider a vector $\mathbf{z} = (z_1, z_2, \dots, z_l)$ such that $z_3 < 0$, $z_1 > 0$, $z_2 > 0$. Let $\mathbf{z}'$ be defined by,

$$\mathbf{z}' = \mathbf{z}_i \quad i \geq 3$$

$$z'_1 = z'_2 = \frac{1}{2}z_1 + \frac{1}{2}z_2.$$

Clearly the positive and negative aggregates of $\mathbf{z}'$ are still Z^+ and Z^- respectively.

Applying (iii) we have that

$$\bar{\phi}_1(\mathbf{z}', Z^+, Z^-) + \bar{\phi}_2(\mathbf{z}', Z^+, Z^-) = \bar{\phi}_1(z_1, Z^+, Z^-) + \bar{\phi}_2(z_2, Z^+, Z^-).$$

Applying (iv) and writing the common function $\bar{\phi}_i$ without the index i , we have

$$\bar{\phi}(\mathbf{z}', Z^+, Z^-) = \frac{1}{2}\bar{\phi}(z_1, Z^+, Z^-) + \frac{1}{2}\bar{\phi}(z_2, Z^+, Z^-).$$

Hence $\bar{\phi}$ is midpoint convex and midpoint concave on the interval from z_1 to z_2 . By choosing z_1 and z_2 arbitrarily on the positive half line and by virtue of (i), $\bar{\phi}$ has these properties throughout the positive half line.

From (v) it can be seen that $\bar{\phi}$ is continuous. Therefore the theorem follows from the proposition in the above remark. Q.E.D.

This theorem characterises the stochastic realisation rules compatible with anonymity, feasibility and the restriction that the functional form depend on aggregates in addition to the agent's own effective demand. The result applies, a fortiori, to non-stochastic rationing rules. Since $s_i(z_i, Z^+, Z^-)$ is a non-random function in this case, it cannot depend on z_i at all.

If we impose the 'short sided' trading rule

$$A \text{ (vi) } z_i \cdot (Z^- + Z^+) < 0 \text{ implies } \phi_i = z_i$$

the theorem can be strengthened to the following,

Corollary

If $\phi_i(z_i, Z^+, Z^-)$ is deterministic and satisfies A (i), A (iii), A (iv), A (v), and A (vi) then

$$\begin{aligned} \phi_i(z'_i, Z^+, Z^-) &= z_i \cdot \min\left(-\frac{Z^-}{Z^+}, 1\right) \text{ if } z_i \geq 0 \\ &= z_i \cdot \min\left(-\frac{Z^+}{Z^-}, 1\right) \text{ if } z_i \leq 0. \end{aligned}$$

Proof

Clearly

$$\begin{aligned} \phi_i &= z_i s^+(Z^+, Z^-) \quad z_i \geq 0 \\ &= z_i s^-(Z^+, Z^-) \quad z_i \leq 0, \end{aligned}$$

where s^+ and s^- are deterministic functions. From (iii)

$$\sum_{(i|z_i \geq 0)} z_i s^+(Z^+, Z^-) = - \sum_{(i|z_i < 0)} z_i s^-(Z^+, Z^-)$$

or

$$s^+(Z^+, Z^-) Z^+ = -s^-(Z^+, Z^-) Z^-.$$

By virtue of (vi),

$$s^+(Z^+, Z^-) = 1 \quad \text{if and only if} \quad Z^+ \leq -Z^-,$$

$$s^-(Z^+, Z^-) = 1 \quad \text{if and only if} \quad Z^+ \geq -Z^-,$$

which, combined with the last statement produces the desired conclusion. Q.E.D.

The model we have used, in which realised trades can be described as functions of effective demand for any of the effective demand in the economy, is not directly comparable to the notion employed by Drèze (1975). In the Drèze system, the quantity constraints perceived by each agent bound his effective demand in an equilibrium. But the system of constraints imposed on an individual when the economy is out of equilibrium is not described. The Drèze definition does not provide a description of how the short-run equilibrium might be attained through 'disequilibrium' quantity adjustments.¹ The theory of resolution of incompatible effective demands presented above does admit such a procedure.²

The situation with respect to the Clower demands is rather different. Effective demands in that framework are derived from explicit rationing rules. If they are not of the proportional form given in the corollary, one of our hypotheses must be violated. Assumptions A (i)-A (v) are standard in their models. Therefore the difference must lie in the fact that they allow ϕ_i to depend on the other agents' effective demands more generally than through aggregates. One can of course prefer the more general dependence to the more specific. Without an explicit theory about the mechanism through which trades among diverse agents are actually consummated, it is impossible to attach specific significance to one functional form or another. Nevertheless, the idea that aggregate market conditions are crucially important in the determination of a short-run equilibrium seems appealing. The conclusion derived on this basis is particularly strong. Moreover, in the context of Clower demands our theorem contradicts

¹ However, such an adjustment process can be constructed; for example, the existence proof given in Grandmont (1977) gives one implicitly.

² For example, agents might believe that the state of the market at time t is $Z_t^+(t)$, $Z_t^-(t)$ and formulate their effective demands $z_i(t)$ accordingly. These effective demands would define the true $z^+(t)$, $z^-(t)$. One might then suppose that, for example,

$$\frac{dZ_t^+(t)}{dt} = \alpha[Z^+(t) - Z_t^+(t)], \quad \frac{dZ_t^-(t)}{dt} = \alpha[Z^-(t) - Z_t^-(t)],$$

and trace out the intra short-period dynamics accordingly. Although we shall not pursue this avenue of research herein (but shall, following Drèze as well as others, presume that a short-period equilibrium is attained without delay) it is useful to observe that the theory presented here is compatible with such a project.

the existence of non-manipulable rationing schemes. This can be seen straightforwardly by noting that the scheme

$$\begin{aligned}\phi_i &= \max[\bar{Z}_i, Y^+(Z^+, Z^-)] \quad (z_i \geq 0) \\ &= \min[\bar{Z}_i, Z^-(Z^+, Z^-)] \quad (z_i \leq 0),\end{aligned}$$

where $\bar{Z}^+$ and $\bar{Z}^-$ are the upper and lower trade bounds is simply not of the form required by the corollary except when $\bar{Z}$ is identically zero. The stochastic rationing scheme proposed by Svensson (1977) as the basis for a theory of effective demand is a straightforward generalisation of the above to the case in which $\bar{Z}^+$ and $\bar{Z}^-$ are allowed to be random functions of Z^+ and Z^- . Thus, if non-manipulable rationing functions are to be consistent with (non-trivial, positive-trade) short-run equilibrium, they must depend on statistics of the effective demands in the population other than the positive and negative aggregates.

One might imagine restrictions on the functional form of the stochastic rationing process more general than dependence upon aggregates alone. These can range from arbitrary dependence on the distribution of the z_i to dependence upon a small number of statistics of this distribution. We have not been successful to date in characterising rationing functions compatible with such generalised functional forms.

II. EXAMPLES OF REALISATION SCHEMES

In order to develop further insight into the implications of the theorem, we present several examples. These are designed to show the dependence of the rationing mechanism on the nature of the underlying matching process. Specifically, in examples 1 and 2, it will be seen that very natural matching processes may not imply rationing mechanisms of the linear form specified in the theorem. We can conclude that they must violate one of the axioms, in particular that dependence on effective demands other than through aggregates is implied. Example 3 will show that the conditions of the theorem can be satisfied in large systems, where individuals' demands are negligible when compared to the aggregate. Thus the theorem might have increased relevance to macroeconomics such as in models of large labour markets with imperfect matching.

Example 1

The rationing mechanism proceeds by placing the agents in a random order and, beginning at the top of this sequence, pairing effective demands and supplies to the maximum possible extent at each step. Once paired off, the trading partners are not dissociated at a later step in the matching process. The random order induces a distribution of realised trades for each agent, as a function of the effective demands in the market.

Suppose there are three agents. The first is a supplier with $z_1 = -2$. The second and third are demanders. Let us compute the distributions realised by these agents as z_2 and z_3 vary in such a way that $z_2, z_3 \geq 0, z_2 + z_3 = +3$.

Let us consider two cases:

- (A) $z'_2 = 1, \quad z'_3 = 2,$
 (B) $z'_2 = 1\frac{1}{2}, \quad z'_3 = 1\frac{1}{2}.$

In case A, the distribution of ϕ_2 is concentrated on $+1$ and 0 with equal probability. In case B it is concentrated on $+1\frac{1}{2}$ and $+1$ with equal probability. Thus, if the conditions of the theorem are fulfilled, we must have

$$E\phi_2 = \frac{1}{2} = 1 \cdot E_{Z_2}(1, 3, 2) \quad \text{in case A}$$

and

$$E\phi_2 = 1 = 1\frac{1}{2} E_{Z_2}(1\frac{1}{2}, 3, 2) \quad \text{in case B.}$$

This is clearly a contradiction of the fact that E_{Z_2} must be independent of the value of z_1 within the positive half-line. The implication is that this rationing rule cannot be described as a random function of the aggregates of supply and demand. The precise distribution of effective demand is necessary to describe the realisation experienced by each agent.

Example 2

Building on Example 1, we consider the case in which each agent's effective demand is divided into k equal pieces. (This might apply to annual demands presented to the market sequentially, or to demands that are parcelled out to several suppliers, as for example in credit markets.) The k pieces are then randomly ordered and the procedure is, as above, to fulfil demands sequentially to the maximum extent possible. For each value of k , the process has the same properties as Example 1. In the limit, however, as $k \rightarrow \infty$, several interesting things happen. Each agent's demand becomes approximately uniformly dispersed in the sequence. As a result of this, the allocation received by each agent converges to

$$z_i \min \left(-\frac{Z^-}{Z^+}, 1 \right) \quad \text{if } z_i \geq 0,$$

$$z_i \min \left(-\frac{Z^+}{Z^-}, 1 \right) \quad \text{if } z_i \leq 0.$$

Note that this is precisely the rationing mechanism of the corollary – that is, it is the only one possible in a deterministic scheme with the short-sided trading rule (v) imposed. The mechanism approached asymptotically in this example does have the characteristic of depending only upon aggregates.

Example 3

If one redefines Z^+ and Z^- to be the per capita positive and negative excess demands in the system, instead of the aggregates, the theorem and corollary, which are stated for a system with fixed number of agents, remain valid. This interpretation is particularly useful in a large economy where z_i is negligible in comparison with Z^+ and Z^- . Such a specification is of particular interest because it is compatible with the assumption that agent i neglects his influence

of Z^+ and Z^- when optimising his choice of effective demands, an assumption that we will use extensively below. A formal treatment can be given using the idealisation of an atomless measure space of economic agents and treating Z^+ and Z^- as the positive and negative parts of mean effective demand.

For this system we can think of the agents having their effective demands ordered as in example 1. Their realisations will then be random variables concentrated on the two points z_i and 0, with probabilities

$$\min\left(-\frac{Z^-}{Z^+}, 1\right) \quad \text{and} \quad \min\left(\frac{Z^+}{-Z^-}, 1\right)$$

for z_i positive and negative respectively. In this example the hypotheses of the theorem are satisfied and the realisations are linear random functions of z_i as required.

Note that the realisations achieved in this way are compatible with the 'short-sided rule', (vi), in addition to the other hypotheses. Somewhat more complex examples can be constructed in which (i)-(v) are satisfied but (vi) is not.

III. INDIVIDUAL BEHAVIOUR

In this section we briefly analyse the implications of the stochastic rationing process we have studied above for the behaviour of individual demands. This would be useful for any equilibrium analysis that would be conducted under these assumptions.

A theory of short-run equilibrium behaviour might utilise the following assumptions on the random realisation functions $\mathcal{Z}_{in}^+$ and $\mathcal{Z}_{in}^-$. They are in the spirit of the decentralisation of resource allocation in different markets.

A (vii) $\mathcal{Z}_{in}^+$ and $\mathcal{Z}_{in}^-$ are independent across distinct markets, n .

A (viii) For Z_n^+ , Z_n^- non-zero, the convex hulls of the supports of $\mathcal{Z}_{in}^+$ and $\mathcal{Z}_{in}^-$ are $(0, 1)$, for all choices of z_{in} .

The justification for condition (vii) is that in a monetary economy in which trade in different markets takes place simultaneously an agent's degree of success in completing his transactions in one, should not affect that in another. It can be greatly weakened without upsetting the validity of our results. Indeed, as long as the realisations are not functionally related across markets there will not be a problem.

Condition (viii) expresses the idea that the short-sided rule is unlikely to hold in an economy where the trading process takes place quickly and there are many agents. The probability that any one agent's demand (supply) might go totally unfulfilled is positive, but so is the probability that the agent is able to transact his entire stated demand (supply). The role of this condition in an equilibrium theory is to bound the effective demands above because the agent risks being unable to fulfil his budget if he is allocated the full stated demand. In the case of supply, it is the fixed physical quantity of the good at his disposal that would bound the effective supply. Such bounds are important for obtaining a result on the existence of an equilibrium.

The agent's problem is

$$\max E u(-p \cdot x_i + M_i, x_i + \omega_i),$$

where

$$x_i = (x_{i1}, \dots, x_{iN}),$$

$$p = (p_1, \dots, p_N) \text{ is a fixed, positive price system,}$$

$$x_{in} = z_{in} \mathcal{Z}_{in}^+(z_{in}, Z_n^+, Z_n^-) \quad z_{in} \geq 0,$$

$$x_{in} = z_{in} \mathcal{Z}_{in}^-(z_{in}, Z_n^+, Z_n^-) \quad z_{in} \leq 0,$$

$$w_i = (\omega_{i1}, \dots, \omega_{iN}) \text{ is a non-negative vector of endowments,}$$

and

$$M_i > 0 \text{ is an initial stock of money,}$$

by choosing $(z_{i1}, \dots, z_{iN})$. The agent is assumed to know, or to have rational expectations about, the aggregate effective demands Z_n^+ , Z_n^- . The domain of definition of u is the non-negative orthant, R_+^{N+1} . Thus, actions that give rise to a positive probability of bankruptcy have undefined expected utilities and are therefore inadmissible.

Theorem

Under conditions (i)-(v), (vii) and (viii), for Z_n^+ , Z_n^- non-zero, the agent's optimal effective demand for each good n is contained in the set bounded by $(-\omega_{in}, M_i/p_n)$.

Proof

This is a direct translation of the feasibility conditions since, even if good n is the only one for which his effective demand is positive, he cannot risk the possibility that none of his effective supplies will be realised at a positive level and that his demands will be realised completely. Q.E.D.

The important characteristic of this theorem, which is an immediate consequence of (viii) is that the bounds on effective demands can be set independently of Z_n^+ , Z_n^- , as long as these are non-zero. This property persists even if (viii) is weakened to the case of any non-degenerate distributions for $\mathcal{Z}_{in}^+$ and $\mathcal{Z}_{in}^-$ containing intervals (α_n^+, β_n^+) and (α_n^-, β_n^-) in their supports for all values of z_{in} in their respective ranges. The corresponding bounds of effective demands would be obtained as follows: For each n , the feasibility condition $\omega_{in} + x_{in} \geq 0$ implies that $z_{in} \geq -\omega_{in}/\beta_n^-$. To obtain the upper bound on z_{in} , suppose that commodity n is the only one for which the individual's effective demand is positive, and that on all other markets his supply is at the maximum calculated above. The greatest lower bound on his income from sales plus initial money holdings, is

$$Y_{in} = M_i + \sum_{n' \neq n} p_{n'} \omega_{in} (\alpha_{n'}/\beta_{n'}).$$

Therefore his maximum permissible effective demand on market n is Y_{in}/β_n^+ .

When either Z_n^+ or Z_n^- are zero, the feasibility condition (iii), combined with the sign-preserving condition (ii), imply that the trades of all agents are identically zero. The stochastic variation in trades required in the last theorem and

the remarks above cannot obtain. However, the agent's allowable effective demands can be truncated in this case, because their realisation of zero is insensitive to the effective demand and they state. No optimal effective demands will be lost in this process.

Returning to the agent's problem, we assume that when maximising his utility the agent assumes that the aggregates Z_n^+ and Z_n^- are constants, unaffected by his own effective demands. Even if U is concave, the solution to this problem will not necessarily be continuous in the positive and negative aggregates (Z_n^+, Z_n^-) , $n = 1, \dots, N$, because the objective function may be non-concave in the z_{in} . These effective demands enter the solution in two ways: multiplicatively through the realisations, and also through their effect on the higher moments, but not the mean of the realisation distribution per unit of effective demand. The second of these effects can result in non-concavities unless further restrictions are placed on the stochastic nature of the realisation process.

Nevertheless we can state the following consequence of the continuity of the maximand in the z_{in} and the previous theorem.

Theorem

The optimal effective demand correspondences $z_{in} = \xi_{in}(Z_n^+, Z_n^-)$ are upper hemi-continuous.

IV. EQUILIBRIUM

Because of the dependence of the stochastic rationing rule ϕ_{in} on z_{in} we cannot assert the convexity of the set of optimising responses, ξ_{in} . This precludes a direct appeal to the fixed-point theorems that are typically used in proving the existence of equilibria. As is well-known, however, in large economies the convexity and continuity of demand correspondences need not be assumed. One either deals with the abstraction of a non-atomic measure space of consumers or seeks only approximate equilibria.

In this framework there is no problem in proving the existence of *some* equilibrium, for it is clear that $Z_n^+ = Z_n^- = 0$ always satisfies the requirements. The existence of an equilibrium with positive levels of trade is a delicate question.¹ Simple hypotheses of monotonicity of preferences will not suffice. Gale (1977) has introduced some restrictions on the rationing schemes under which a non-trivial (positive-trade) equilibrium is sure to exist. Because Gale's theorem is very general in its treatment of stochastic rationing rules it remains an open question as to whether his conditions can be satisfied by stochastic rationing schemes of the particular form studied in this paper.²

Harvard University

JERRY GREEN

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¹ In a model with autonomous demand from a government sector, and with all other assumptions as in this paper, Honkapohja and Ito (1978) have proved the existence of an equilibrium with positive trades by direct appeal to the structure of demands we have derived above. The autonomous demand precludes no-trade equilibria.

² Bohm and Levine (1977) and Heller and Starr (1977) have studied the existence of equilibria with a positive level of trade under the Nash equilibrium hypothesis for deterministic trading rules.

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